FYSS9000, High energy scattering in QCD, spring 2023

Exercise 4, tutorial session Mon June 6th at 10-12 (round table room), return by Wed June 7th at 18.00.
May 23: corrected equation for Bessel asymptotics and moved hint to right place

1. Consider two (independent of each other) transverse ($i, j \in \{1, 2\}$) pure gauge fields that depend only on transverse coordinates $A_i^{(1,2)} = A_{i,a}^{(1,2)} t^a = \frac{-i}{g} V(\mathbf{x}_T)\partial_i V^\dagger(\mathbf{x}_T)$. Recall the expression for the field strength tensor $F_{\mu\nu}$ and show that these pure gauges have no longitudinal magnetic field $F_{ij}^{(1,2)} = 0$. Then consider a field that is the sum of the two: $A_i = A_i^{(1)} + A_i^{(2)}$: what is its magnetic field F_{ij}?

2. Express the field strength tensor components corresponding to the normal t, x, y, z coordinates in terms of the tensor in $\tau, \eta, \mathbf{x}_T$-coordinates (such as $F_{\tau\eta} = \partial_\tau A_\eta$ etc.). In other words, find $F_{ti}, F_{tz}, F_{iz}, F_{ij}$ with $i, j = 1, 2$ in terms of $F_{\tau\eta}, F_{\tau i}, F_{i\eta}$. Remember that $\eta = \frac{1}{2}\ln x^+/x^-$ and $\tau = 2x^+x^- = \sqrt{t^2 - z^2}$ and that $F_{\mu\nu}$ transforms as a Lorentz-tensor, i.e. $F_{\mu'\nu'} = (\partial_{\mu'} x^\mu)(\partial_{\nu'} x^\nu)F_{\mu\nu}$.

3. Let us calculate the spatially averaged energy-momentum tensor of a boost invariant **Abelian** field (at zero rapidity). Taking the vector potential as (with the Levi-Civita tensor $\varepsilon_{12} = -\varepsilon_{21} = 1$)

$$A_i(\tau, \mathbf{x}_T) = \int \frac{\mathrm{d}^2\mathbf{k}_T}{(2\pi)^2} \frac{-i\varepsilon_{ij}k^j f(\mathbf{k}_T)}{\mathbf{k}_T{}^2} e^{i\mathbf{k}_T\cdot\mathbf{x}_T} J_0(|\mathbf{k}_T|\tau) \tag{1}$$

$$A_\eta(\tau, \mathbf{x}_T) = \int \frac{\mathrm{d}^2\mathbf{k}_T}{(2\pi)^2} f(\mathbf{k}_T) e^{i\mathbf{k}_T\cdot\mathbf{x}_T} \frac{\tau}{|\mathbf{k}_T|} J_1(|\mathbf{k}_T|\tau) \tag{2}$$

calculate first $F_{\tau\eta}, F_{\tau i}, F_{i\eta}, F_{ij}$. Then, using the result of the previous problem (setting $\eta = 0$, $z = 0$) obtain F_{ti}, F_{tz}, F_{iz} and F_{ij} (i.e. $F_{xy} = -F_{yx}$). Note that

$$\frac{\mathrm{d}}{\mathrm{d}z} J_0(z) = -J_1(z) \quad \frac{\mathrm{d}}{\mathrm{d}z}(zJ_1(z)) = zJ_0(z) \tag{3}$$

Then from these calculate

$$\int \mathrm{d}^2\mathbf{x}_T T_{00} = \int \mathrm{d}^2\mathbf{x}_T \frac{1}{2}\left(F_{tx}^2 + F_{ty}^2 + F_{tz}^2 + F_{xz}^2 + F_{yz}^2 + F_{xy}^2\right) \tag{4}$$

$$\int \mathrm{d}^2\mathbf{x}_T T_{xx} = \int \mathrm{d}^2\mathbf{x}_T \frac{1}{2}\left(F_{ty}^2 + F_{tz}^2 - F_{tx}^2 + F_{xz}^2 + F_{xy}^2 - F_{yz}^2\right) \tag{5}$$

$$\int \mathrm{d}^2\mathbf{x}_T T_{yy} = \int \mathrm{d}^2\mathbf{x}_T \frac{1}{2}\left(F_{tx}^2 + F_{tz}^2 - F_{ty}^2 + F_{yz}^2 + F_{xy}^2 - F_{xz}^2\right) \tag{6}$$

$$\int \mathrm{d}^2\mathbf{x}_T T_{zz} = \int \mathrm{d}^2\mathbf{x}_T \frac{1}{2}\left(F_{tx}^2 + F_{ty}^2 - F_{tz}^2 + F_{xz}^2 + F_{yz}^2 - F_{xy}^2\right) \tag{7}$$

4. Check the values at $\tau = 0$ and $\tau \to \infty$ of components of the energy momentum tensor using the asymptotic behavior of the Bessel functions: $J_0(z) \underset{z\to\infty}{\to} \sqrt{\frac{2}{\pi z}}\cos(z - \pi/4)$ and $J_1(z) \underset{z\to\infty}{\to} \sqrt{\frac{2}{\pi z}}\cos(z - 3\pi/4)$. Do you see how the anisotropic gluon momentum distribution, i.e. $T_{xx} \sim T_{yy} \gg T_{zz}$ arises?

5. What happens if we neglect the LPM effect in the last (energy loss) stage of bottom-up thermalization scenario? That is, if instead of (5.107) the emission rate is just given by the Bethe-Heitler rate (cross section from (5.96) times the number density T^3, with $m_D^2 \sim \alpha_\mathrm{s} T^2$)

$$\frac{1}{t_\mathrm{br}} \sim \alpha_\mathrm{s}^2 T. \tag{8}$$

At what time $Q_\mathrm{s}\tau \sim \alpha_\mathrm{s}^?$ have the hard modes lost all their energy? Would this estimate be consistent with the end previous stage of the bottom-up scenario? (Note that doing this we are assuming that the density of scattering centers $\sim T^3$ and the Debye mass $m_D^2 \sim \alpha_\mathrm{s} T^2$ are determined by the soft particles, they are the ones that form a system with temperature T).

6. Find at least one typo in the latest version of the lecture note. Alternatively: identify an equation in the lecture note that is not explained clearly and explain what is not clear about it.