FYSS9000, High energy scattering in QCD, spring 2023

Exercise 3, tutorial session Mon May 8th at 10-12 (round table room), return by Wed May 10th at 18.00.

1. At HERA, a 27.5GeV electron beam collides with a 920GeV proton beam. In one event, $x = 0.1$ and $Q^2 = 50\text{GeV}^2$. In another, $x = 0.001$ and $Q^2 = 3\text{GeV}^2$. What is the scattering angle (with respect to the incoming electron beam) and energy of the outgoing electron in these events? You can neglect the proton mass.

2. In the target rest frame the electron-proton cross section is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E'\mathrm{d}\Omega} = \frac{\alpha_{\text{em}}^2}{2mQ^2}\frac{E'}{E}L_{\mu\nu}W^{\mu\nu},$$

where the leptonic tensor is $L_{\mu\nu} = 2(k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k\cdot k')$ and the hadronic tensor

$$W_{\mu\nu} = -2\left(g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2}\right)F_1(x,Q^2) + \frac{2}{P\cdot q}\left[\left(P_\mu + \frac{P\cdot q}{Q^2}q_\mu\right)\left(P_\nu + \frac{P\cdot q}{Q^2}q_\nu\right)\right]F_2(x,Q^2) \tag{1}$$

Calculate $L_{\mu\nu}W^{\mu\nu}$ in terms of the Lorentz invariants $x, y, Q^2 = -q^2$ and m_N^2 (the electron is massless).

3. Let us introduce the projection operators to the virtual photon polarization states as

$$d_{\mu\nu} = -\left(g_{\mu\nu} + \frac{q^\mu q^\nu}{Q^2}\right) = -\varepsilon_\mu^{\text{L}}(q)(\varepsilon_\nu^{\text{L}}(q))^* + \sum_{\lambda=\pm1}\varepsilon_\mu^{(\lambda)}(q)(\varepsilon_\nu^{(\lambda)}(q))^* \tag{2}$$

$$d_{\mu\nu}^{\text{L}} = \frac{Q^2}{m^2(\nu^2+Q^2)}\left(P_\mu + \frac{P\cdot q}{Q^2}q_\mu\right)\left(P_\nu + \frac{P\cdot q}{Q^2}q_\nu\right) \tag{3}$$

$$d_{\mu\nu}^{\text{T}} = \frac{1}{2}\left(d_{\mu\nu} + d_{\mu\nu}^{\text{L}}\right) \tag{4}$$

These projectors have slightly unconventional properties from a mathematical point of view, due to the nonpositive Lorentz metric and the fact that we include the average over the two transverse spins in the projector. Note that there is a sign confusion in Barone & Predazzi. Using these projectors and the decomposition of $W_{\mu\nu}$ given above in Eq. (1) express

$$\sigma_{\text{L,T}}^{\gamma^* p} = \frac{4\pi^2\alpha_{\text{e.m.}}}{Q^2(1-x)}x d_{\mu\nu}^{\text{L,T}}W^{\mu\nu}$$

in terms of F_1 and F_2. Here we are (hopefully correctly) using the Hand convention for the flux factor $4m\nu(1-x)$ as in Halzen & Martin, differently from the Gilman convention in Barone & Predazzi. In the lecture note the proton mass terms and the x in the flux factor are neglected, but they are easy to include here. Do you recover the expression used in eq. (10) of hep-ex/9510009 (ZEUS collaboration).

4. Calculate

$$\int_{-\infty}^{\infty}\mathrm{d}k\,\frac{e^{ikx}}{k^2+m^2}.$$

This can be done the easy way using the theorem of residues; there is probably also a hard way without it. Then calculate the two dimensional transform needed in the virtual photon-dipole wave function

$$\int \mathrm{d}^2\mathbf{k}_T\,\frac{e^{i\mathbf{k}_T\cdot\mathbf{r}_T}}{{\mathbf{k}_T}^2+m^2}$$

and compare the behavior at large $r = |\mathbf{r}_T|$. *One way is to integrate over the angle first, which leaves you with the Bessel $J_0(|\mathbf{k}_T||\mathbf{r}_T|) = J_0(kr)$. The integral over k then gives a modified Bessel K_0. Mathematica will do this k-integral for you. The way to do this without mathematica is to do a Schwinger parametrization of the denominator $1/A = \int_0^\infty \mathrm{d}t e^{-tA}$, then do the Gaussian integral over $\mathbf{k}_T$ and then recognize an integral representation of the Bessel K_0 (see e.g. 10.32.10 in https://dlmf.nist.gov/10.32).*

5. Fourier-transform the essential part of the LC wave function for emitting a soft gluon:

$$\int \mathrm{d}^2\mathbf{k}_T e^{i\mathbf{k}_T\cdot\mathbf{r}_T} \frac{\boldsymbol{\varepsilon}_T \cdot \mathbf{k}_T}{\mathbf{k}_T{}^2}.$$

This can be done analytically (without mathematica!) by first integrating over the angle, which gives a Bessel function J_1 that is the derivative of J_0; thus the radial integral is easy. Note that there are two independent azimuthal angles, those of $\boldsymbol{\varepsilon}_T$ and $\mathbf{r}_T$.

6. (Kovchegov & Levin, 4.5 b) Solve the BK equation in zero transverse dimensions:

$$\partial_y N = \alpha_\mathrm{s} N - \alpha_\mathrm{s} N^2, \quad N(y=0) = N_0 \ll 1 \tag{5}$$