FYSS9000, High energy scattering in QCD, spring 2023

Exercise 1, tutorial session Mon April 3rd (round table room), return by Wed April 12th at 18.00.

1. The differential cross section for the $2 \to n$ process is

$$\mathrm{d}\sigma = \frac{1}{2s} |A(i \to f_n)|^2 \,\mathrm{d}\Pi_n,$$

where $\mathrm{d}\Pi_n$ is the n-particle invariant phase space.

   (a) What is the dimensionality (in GeV) of the scattering amplitude A for an arbitrary n?

   (b) Consider the following processes: $qq \to qq$, $gg \to gg$, $qq \to qqgg$ and $gg \to ggg$ (q is a quark and g is a gluon). Draw at least one Feynman diagram contributing to each, add up the dimensionalities of the factors from the external lines, propagators and vertices and check that the amplitude has the right dimensions.

2. Let's practice SU(N_c) algebra. The fundamental representation is generated by the traceless Hermitian $N_\mathrm{c} \times N_\mathrm{c}$ matrices t_a, $a = 1 \dots N_\mathrm{c}^2 - 1$ normalized as $\mathrm{Tr} t_a t_b = \frac{1}{2}\delta_{ab}$. Their commutator is $[t_a, t_b] = i f_{abc} t_c$ with f_{abc} antisymmetric in all three indices (this actually follows from the chosen normalization). The fundamental representation Casimir operator is $t_a t_a = C_\mathrm{F} \mathbb{1}_{N_\mathrm{c} \times N_\mathrm{c}}$. What is C_F ? The adjoint representation is generated by $(T_a)_{bc} = -i f_{abc}$. The adjoint Casimir C_A in $f_{abc} f_{abd} = C_\mathrm{A} \delta_{cd}$ is $C_\mathrm{A} = N_\mathrm{c}$ (can you show this?). Using these calculate $\mathrm{Tr}(t_a t_b t_a t_b)$.

3. Calculate (to lowest order, one tree diagram) the cross section for fermion-fermion (e.g. $e^-\mu^- \to e^-\mu^-$) scattering in the Yukawa interaction, i.e. via a scalar particle. The Feynman rule for the fermion-scalar particle vertex is just g (the coupling), with spinors for the external legs, and the scalar particle propagator is just i/q^2. What is the high energy limit? (This calculation is shorter and simpler than in Sec 2.2 in the lecture note).

4. Take the Feynman rules for the graviton propagator (160) and the graviton-spin-0 particle vertex (162) from arXiv:1702.00319. Calculate the elastic scattering cross section $\mathrm{d}\sigma/\mathrm{d}t$ between two unidentical massless spin-0 particles via the exchange of a graviton (just one diagram, similar to the previous problem). What is the dimension of the coupling κ so that the cross section has the right dimensions? What is the high energy behavior $s \approx -u \gg -t > 0$?

5. (a) Show (this is easy) that if

$$\frac{\mathrm{d}\sigma_{\mathrm{el.}}}{\mathrm{d}^2\mathbf{q}_T} = \left| \frac{i}{2\pi} \int \mathrm{d}^2\mathbf{b}_T e^{-i\mathbf{q}_T \cdot \mathbf{b}_T} \Gamma(\mathbf{b}_T) \right|^2$$

   then

$$\sigma_{\mathrm{el}} = \int \mathrm{d}^2\mathbf{b}_T |\Gamma(\mathbf{b}_T)|^2$$

   (b) The total cross section is

$$\sigma_{\mathrm{tot}} = 2 \int \mathrm{d}^2\mathbf{b}_T \mathrm{Re}[\Gamma(\mathbf{b}_T)],$$

   and the partial wave unitarity bound is $|\Gamma(\mathbf{b}_T)|^2 \leq 2\mathrm{Re}[\Gamma(\mathbf{b}_T)]$, which leads to $\sigma_{\mathrm{el}} \leq \sigma_{\mathrm{tot}}$ (a pretty natural requirement). Where in the complex plane can $\Gamma(\mathbf{b}_T)$ be to satisfy this?

(c) Assuming that $\mathrm{Re}[\Gamma(\mathbf{b}_T)] = \Gamma_0 e^{-\mathbf{b}_T^2/(2B)}$ and $\mathrm{Im}[\Gamma(\mathbf{b}_T)] = 0.141\mathrm{Re}[\Gamma(\mathbf{b}_T)]$, $\sigma_{\mathrm{el}} = 25.4\mathrm{mb}$ and $\sigma_{\mathrm{tot}} = 98.6\mathrm{mb}$, what are B and Γ_0 ? The cross section numbers for pp-scattering at $\sqrt{s} = 7\mathrm{TeV}$ come from TOTEM, Europhys.Lett. 101 (2013) 21002, `https://cds.cern.ch/record/1472948/files/CERN-PH-EP-2012-239.pdf`. Is the Gaussian $\mathbf{b}_T$-dependence consistent with Fig. 2 of the paper? ($t = -\mathbf{q}_T^2$)

6. Look at: Phys.Lett. B687 (2010) 174; `arXiv:1001.1378 [hep-ph]` What are $\bar{N}_F$ and $\bar{N}_A$ in eq (1)? Derive Eq. (6) in the paper using color identities that you can find e.g. in the lecture note. Note the large N_c limit.