1. The time dilation has been experimentally tested for example by flying atomic clocks in airplanes around the Earth. In this case the time dilation is actually affected by special and general relativity effects but let us consider only the special relativity contribution here.

   a) The radius of the Earth at equator is $R = 6378\text{km}$. An airplane flies above the equator at the altitude $h = 9\text{km}$ and its velocity with respect to the surface of Earth is $v = 300\text{m/s}$. What is the time T during which the airplane completes one full orbit around the Earth, as measured from an (approximative) inertial frame fixed at the center of the Earth?

   b) The expression for proper time $d\tau = dt\sqrt{1 - \frac{1}{c^2}|\frac{d\bar{x}}{dt}|^2}$ applies also for accelerated motion. For an object moving with a constant angular velocity ω on a circular orbit $\bar{x} = (R\cos(\omega t), R\sin(\omega t), 0)$, we have

   $$d\tau = dt\sqrt{1 - \frac{R^2\omega^2}{c^2}} \ .$$

   A clock A is at rest with respect to the surface of the Earth with $R_A = R$ ja $\omega_A = 2\pi/(24\text{h}) = 7.27 \times 10^{-5}\text{s}^{-1}$. Another clock is at rest in the airplane of case a, for which $R_B = R + h$ and $\omega_B = \omega_A R_A/R_B + v/R_B$ (flight along the Earth's direction of rotation). What is the time difference measured by the clocks A and B for one full orbit around the Earth?

2. What is the kinetic energy of an electron ($m_e = 511\text{keV}/c^2$) moving at $v = 0.8c$? How much does the energy change when the velocity is increased to $v = 0.9c$?

3. An inertial observer O measures the momentum $|\bar{p}| = 817$ MeV/c and energy $E = 1125$ MeV for an elementary particle. An observer O' moving at a constant velocity with respect to O measures the momentum $|\bar{p}'| = 953$ MeV/c for the same particle.

   a) What is the mass of the particle?

   b) What is the energy of the particle as measured by O'?

4. The mass of uranium atom ^{235}U is $m \approx 235\ u$, where $u = 931.5$ MeV/c^2. In a fission reaction, 200 MeV of the mass energy is transferred into the kinetic energy of the decay products. How much energy is released when $m = 1.00\text{kg}$ of uranium decays through the fission reaction, and what is the mass change in the process?

5. A pion π^- decays to a muon μ^- and to the muon's antineutrino $\bar{\nu}_\mu$. The mass of the pion is $m_\pi = 1.4\, m_\mu$ and the mass of the antineutrino can be neglected here.

   a) Write down the expressions for energy and momentum conservation in the center of mass frame (pion's rest frame).

   b) Compute the energy of the muon generated in the process and compare the result to the rest energy of the muon. Would the process be possible if $m_\pi = 0.9\, m_\mu$?