1. An elementary particle created in a bubble chamber moves at constant velocity $v = 0.995c$ and travels a distance 1.25 mm before decaying. What is the lifetime of the particle in its rest frame?

2. A rocket moves at constant velocity v from $(x_A, t_A) = (0, 0)$ to $(x_B, t_B) = (4.32 \times 10^{11}\text{m}, 3600\text{s})$ in an inertial frame K.

   a) What is the time interval $\Delta t'$ measured by the rocket's pilot between the spacetime events A and B?

   b) What is the distance that the rocket's pilot should measure for $x_B - x_A$ according to the Lorentz contraction formula?

   c) Compare the above result to $v\Delta t'$.

3. Two physics students intend to commence enjoying their Friday evening refreshments (lemonade) simultaneously. Having synchronised their clocks, they head to different refreshment premises separated by a distance of 400 meters. Unfortunately, the clock synchronisation fails, resulting in a time difference of 2×10^{-7} seconds between the onsets of their lemonade consumption. To end the ridicule of the mathematicians following the process, the physicists proclaim that simultaneity is relative. Determine the velocity at which an inertial observer needs to move with respect to the refreshment premises in order for the onsets of the lemonade consumption to appear simultaneous in that frame.

4. A painfully slow lecture starts at $t = 0$, after which the slides change at regular intervals of $\Delta t = 3$ minutes. At $t = 15$ min, a student sitting in the back row is struck by an imminent threat of falling asleep, and decides to escape the lecture hall in a rocket moving at velocity $v = 0.7c$. Unfortunately, the door of the lecture hall is left open so that the slides remain visible in the rocket's rearview mirror. To their horror, the studend realises that due to the Doppler effect, the lecture appears to be proceeding even more slowly as observed from the rocket.

   a) What is the interval between slides $\Delta t'$ as observed by the student in the rocket?

   b) The lecture ends at $t = 45$ min. How long does it take until the information of this event of relief reaches the student?

5. In special relativity, the spacetime distance between differentially separated points (t, x, y, z) and $(t + dt, x + dx, y + dy, z + dz)$ is defined as

$$ds^2 \equiv c^2 dt^2 - dx^2 - dy^2 - dz^2 \ .$$

   a) Show that ds^2 remains invariant under Lorentz transformations.

   b) Consider motion in the (t, x) plane so that $dy = dz = 0$. What is the velocity of objects moving along trajectories with $ds^2 = 0$? What can you say about the velocities along trajectories with $ds^2 > 0$ and $ds^2 < 0$?

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6. Consider the setup described on page 30 of the lecture notes. Matti stays at rest in Turku and Teppo heads towards Sirius with a rocket whose veloctity with respect to Matti is $v = 0.8c$. At $t = 0$ in Matti's rest frame (unprimed coordinates (x, t)), Matti and Teppo are located at the same spacetime point $(x_\mathrm{A}, t_\mathrm{A}) = (0, 0)$.

   a) Teppo celebrates the one-year anniversary of his journey at $(x'_\mathrm{B}, t'_\mathrm{B}) = (0, 1\mathrm{y})$ in his rest frame coordinates (event B). Compute the coordinates $(x_\mathrm{B}, t_\mathrm{B})$ of the event B in Matti's rest frame. Compare the time difference $t_B - t_A$ against the time dilation formula.

   b) Matti celebrates the one-year anniversary according to his own clock at $(x_\mathrm{C}, t_\mathrm{C}) = (0, 1\mathrm{y})$ in his rest frame coordinates (event C). Compute the coordinates $(x'_\mathrm{C}, t'_\mathrm{C})$ of the event C in Teppo's rest frame. In Matti's rest frame Teppo's coordinates at t_C are $(x_\mathrm{E}, t_\mathrm{E}) = (vt_\mathrm{C}, t_\mathrm{C})$ and in Matti's frame events C and E simultaneous. Are they simultaneous also in Teppo's rest frame? Is there a unique answer to the question: "What is the time interval in Teppo's rest frame that corresponds to one year in Matti's rest frame?"

   c) At point C, Matti sends a message to Teppo. The message travels at the speed of light and reaches Teppo at $(x_\mathrm{D}, t_\mathrm{D})$. Compute the time t'_D, in Teppo's rest frame coordinates, when the message reaches Teppo. Teppo computes the time t'_D0 when the message was sent, according to his rest frame coordinates, by subtracting the travel time from t_D. Compare Teppo's result to the result you found in case b.