Electrical Impedance Tomography: Computation and Applications

Part 7: The Complete Electrode Model

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Jyväskylä Summer School 2026

August 2026

From Measurements to a Forward Model

So far, we've discussed the voltage data V_{meas} that come from physical measurements, and also the continuum model.

Given a candidate conductivity, known electrode geometry and applied currents I, an electrode forward model predicts what the instrument would measure:

$$\sigma(x),\ I \quad \xrightarrow{\text{forward model}} \quad V_{\mathrm{pred}}.$$

A realistic electrode forward model allows us to:

- support linearized and iterative reconstruction,
- generate synthetic data to test reconstruction algorithms and measurement strategies,
- connect ideal continuum data with finite electrode measurements.

Common Boundary and Electrode Models

Different models retain different features of the physical electrodes:

Model	Basic idealization
Continuum model	Prescribe current density and measure voltage directly on the boundary; no discrete electrodes
Point electrode model	Treat electrodes as points; ignore their finite size and contact impedance
Gap model	Use finite electrodes with uniform applied current density, but voltages doesn't need to be constant on an electrode
Shunt model	Treat each finite electrode as a perfect conductor (so voltage is constant), but assume zero contact impedance
Complete electrode model	Include finite, equipotential electrodes, redistribution of current, and nonzero contact impedance

Introducing the Complete Electrode Model

We will focus on the *Complete Electrode Model (CEM)*, which is widely regarded as the most accurate standard model for electrode-based EIT measurements.[1]

Simpler models are easier and cheaper to solve, but the CEM usually gives the most realistic description of practical low-frequency EIT measurements.

What the CEM Predicts

Given the conductivity, electrode configuration, contact impedances, and applied electrode currents, the CEM predicts the resulting electrode voltages.

The model accounts for:

- the finite size and number of the electrodes,
- electrical contact between the electrodes and target,
- gaps between electrodes.

[1] Somersalo, Cheney, and Isaacson, *SIAM J. Appl. Math.* 52 (1992), 1023–1040.

How the CEM Models Applied Current

The CEM takes the *total* current delivered through each electrode as input:

$$\boxed{I_\ell = \int_{e_\ell} \sigma \frac{\partial u}{\partial \mathbf{n}} \, ds.}$$

It does not prescribe the current density separately at every point under the electrode.

- We assume the local current density may vary across the electrode.
- Due to conservation of charge, total electrode currents must satisfy

$$\sum_{\ell=1}^{L} I_\ell = 0.$$

- We assume no current crosses the boundary between electrodes:

$$\sigma \frac{\partial u}{\partial \mathbf{n}} = 0 \qquad \text{on } \partial\Omega \setminus \bigcup_{\ell=1}^{L} e_\ell.$$

Modeling Electrode Contact Imperfections

The voltage V_ℓ of the metal electrode need not equal the tissue-side potential $u(x)$ directly beneath it.

A thin contact layer (e.g. skin surface) creates an additional voltage drop.
Let $z_\ell > 0$ denote the effective contact impedance of electrode e_ℓ. The complete electrode model assumes

$$V_\ell - u = z_\ell \sigma \frac{\partial u}{\partial \mathbf{n}} \qquad \text{on } e_\ell.$$

Equivalently,

$$\boxed{u + z_\ell \sigma \frac{\partial u}{\partial \mathbf{n}} = V_\ell \qquad \text{on } e_\ell.}$$

If $z_\ell = 0$, this reduces to perfect electrical contact: $u = V_\ell$ on the electrode.

Units of the Contact Impedance

On electrode e_ℓ, the CEM imposes

$$u + z_\ell j_n = U_\ell, \qquad j_n = \sigma \frac{\partial u}{\partial \mathbf{n}}.$$

Since j_n is current per unit area,

$$[j_n] = \mathrm{A}/\mathrm{m}^2, \qquad [z_\ell] = \frac{\mathrm{V}}{\mathrm{A}/\mathrm{m}^2} = \boxed{\Omega\,\mathrm{m}^2}.$$

So the CEM parameter z_ℓ is not the ordinary electrode impedance in ohms. For an electrode of area A_ℓ, the corresponding scale is

$$R_{\mathrm{contact}} \approx \frac{z_\ell}{A_\ell}.$$

For example, if $A_\ell = 1\,\mathrm{cm}^2$ and $z_\ell = 10^{-2}\,\Omega\,\mathrm{m}^2$, then $R_{\mathrm{contact}} \approx 100\,\Omega$.

In a 2D model, the out-of-plane height may be absorbed into the parameter, giving units of $\Omega\,\mathrm{m}$ instead.

Typical Contact-Impedance Scales

Values vary greatly with electrode material, skin preparation, and frequency. Some rough physical scales are:

Contact	z_ℓ **in** $\Omega\,\mathrm{m}^2$	z_ℓ / A_ℓ **for** $1\,\mathrm{cm}^2$
Metal electrodes in a saline tank	10^{-4}–10^{-2}	1–100 Ω
Well-prepared, gelled skin contact	10^{-2}–10^{-1}	100 Ω–1 $\mathrm{k}\Omega$
Dry or poor skin contact	10^{-1}–1 or more	1–10 $\mathrm{k}\Omega$ or more

AC Measurements

Contact impedance generally depends on frequency and may be complex. At a fixed drive frequency, the basic CEM often uses an effective positive real value for z_ℓ.

The CEM Equations

Given the conductivity σ, contact impedances z_ℓ, and balanced electrode currents I, solve for the interior potential u and electrode voltages V_{pred}:

$$
\begin{cases}
\nabla \cdot (\sigma \nabla u) = 0 & \text{in } \Omega, \\
\sigma \dfrac{\partial u}{\partial \mathbf{n}} = 0 & \text{between electrodes,} \\
u + z_\ell \sigma \dfrac{\partial u}{\partial \mathbf{n}} = V_\ell & \text{on } e_\ell, \\
\displaystyle\int_{e_\ell} \sigma \frac{\partial u}{\partial \mathbf{n}}\, ds = I_\ell, & \ell = 1, \ldots, L, \\
\displaystyle\sum_{\ell=1}^{L} V_\ell = 0 & \text{(reference choice).}
\end{cases}
$$

This is the “gold standard” model used in most EIT simulation and reconstruction

Using the CEM

Next, we will examine a model-based reconstruction method that uses the CEM in the forward-modeling step.