Electrical Impedance Tomography: Computation and Applications

Part 6: A Survey of Reconstruction Methods

Melody Alsaker

Jyväskylä Summer School 2026

August 2026

From EIT Data to an Image

We now have data from an EIT system:

$$\mathcal{D} = \left\{ \left(\mathbf{I}^{(n)}, \mathbf{V}^{(n)} \right) \right\}_{n=1}^{N}.$$

Reconstruction asks what interior conductivity information produced those measurements.

Absolute imaging	estimate $\sigma(x)$ from one data set $\mathcal{D}$
Time-difference imaging	estimate the change from t_1 to t_2
Frequency-difference imaging	estimate changes across AC frequencies

In every case, we must extract stable information from a nonlinear, severely ill-posed data map.

Why Absolute Imaging Is Hard

Absolute imaging must explain the full measured signal:

$$V_{\mathrm{meas}} \approx F(\sigma;\theta), \qquad \theta = \{\text{geometry, electrodes, contact impedances, calibration}\}.$$

- The map F is nonlinear, and we must recover both the overall conductivity level and its spatial variation.
- The nuisance parameters θ are never known exactly.
- Severe ill-posedness amplifies small measurement errors and model mismatch.

The Central Ambiguity: A data discrepancy may come from the interior conductivity or from an imperfect model of the experiment, and we have no way of knowing which it is.

Useful when: the static conductivity itself matters, or no reliable reference data set is available; locating a brain lesion in a one-time clinical scan, imaging the oil / water distribution in a pipeline, or mapping a subsurface contaminant plume.

Why Time-Difference Imaging Is Usually More Stable

Suppose two data frames are collected with the same acquisition setup:

$$V_1 = F(\sigma_1; \theta) + b + \varepsilon_1, \qquad V_2 = F(\sigma_2; \theta) + b + \varepsilon_2.$$

Here, b represents a stable additive measurement bias, such as an electronic voltage offset, common to both frames. The errors ε_1 and ε_2 may be very similar in size.

Subtracting removes the common offset b and reduces other shared errors:

$$\Delta V = V_2 - V_1 \approx F(\sigma_2) - F(\sigma_1)$$

- We reconstruct only the change, not the entire background.
- Stable offsets cancel, and the effects of other shared systematic errors are often strongly reduced.

Useful for: lung ventilation or perfusion, gastric emptying, and monitoring an intervention over time.

Electrode motion, geometry changes, drift, or a large conductivity change can destroy these advantages.

Frequency-Difference Imaging

Measure the same object at two AC frequencies and compare

$$\Delta V_\omega = V(\omega_2) - V(\omega_1).$$

Different tissues or materials can have different frequency-dependent electrical properties. Frequency differencing emphasizes this *spectral contrast* rather than change over time.

Useful When

The target and background have distinguishable frequency responses; for example, in tissue characterization, stroke-classification research, or distinguishing components of an industrial mixture.

The cancellation is less automatic than in time-difference imaging: electronics, contact impedance, and applied-current amplitude may also vary with frequency. Careful calibration or normalization is essential.

Why Regularization Is Unavoidable

No matter what, though, EIT is severely ill-posed: small errors in the data or forward model can produce large errors in a reconstructed image.

An unregularized inversion can strongly amplify the most unstable, noise-sensitive components of the data.

Regularization

Deliberately limit this amplification: retain information that can be recovered reliably while suppressing components dominated by noise or modeling error.

The usual tradeoff is greater stability at the cost of some spatial detail.

Every practical EIT reconstruction method requires some form of regularization. We will come back to this.

A Map of Reconstruction Approaches

Approach	Basic strategy
One-step model-based	Invert a local linear model at a fixed reference
Iterative model-based	Repeatedly predict the data and update the conductivity
Constructive direct	Transform boundary data into an image using mathematical construction
Testing and sampling direct	Test candidate points and regions for the presence of an anomaly
Bayesian/statistical	Combine a forward model with prior information to infer a posterior distribution for the conductivity
Learning-based	Learn all or part of the data-to-image map

These methods can overlap. Learning and Bayesian ideas can be combined with several of these approaches.

What Are "Model-Based" Methods?

After discretizing the target into pixels or finite elements, let σ denote the resulting conductivity image.

For a fixed measurement protocol, a computational forward model predicts voltages:

$$\underbrace{\sigma}_{\text{conductivity image}} \xrightarrow{\text{forward model } F} \underbrace{F(\sigma)}_{\text{predicted voltage data}} .$$

Model-based reconstruction means using these predictions to seek σ for which

$$F(\sigma) \approx V_{\mathrm{meas}}.$$

Here V_{meas} denotes all retained voltage measurements, stacked in a single vector.

Two Ways to Use the Model

One-step: Build one local approximation near a fixed reference.

Iterative: Repeatedly predict voltages and refine the image.

Linearization

The conductivity-to-voltage map $F(\sigma)$ is nonlinear in σ. Choose a reference image σ_0, often a homogeneous estimate.

Near σ_0, replace the nonlinear change in F by its first-order Taylor approximation:

$$F(\sigma) - F(\sigma_0) \approx \underbrace{J(\sigma_0)}_{\text{Jacobian}} (\sigma - \sigma_0).$$

Each column of $J(\sigma_0)$ describes the voltage change caused by a small conductivity change in one pixel or finite element.

Since $V_{\mathrm{meas}} \approx F(\sigma)$, this gives

$$\underbrace{V_{\mathrm{meas}} - F(\sigma_0)}_{\Delta V} \approx J(\sigma_0) \underbrace{(\sigma - \sigma_0)}_{\Delta\sigma}.$$

One-Step Methods: Linearize Once

Why “One-Step”?

Keep σ_0 and $J(\sigma_0)$ fixed, regularize the linear inversion, and make one conductivity update.

For fixed quadratic regularization, a regularized approximate inverse G_α can be computed in advance:

$$\Delta\sigma = G_\alpha \, \Delta V.$$

Examples: NOSER; one-step Gauss–Newton with Tikhonov or Laplacian regularization; truncated-SVD inversion.

Reconstruction is extremely fast, but the local approximation becomes less accurate farther from the reference.

Iterative Methods: Predict, Compare, Update

Begin with an initial guess σ_0. For $k = 0, 1, 2, \ldots$:

1. **Predict:** solve the forward problem to compute $F(\sigma_k)$.
2. **Compare:** form the residual
$$\mathbf{r}_k = V_{\mathrm{meas}} - F(\sigma_k).$$
3. **Update:** use $\mathbf{r}_k$, sensitivity information, and regularization to construct σ_{k+1}.

Repeat the cycle, often recomputing the Jacobian (sensitivity) matrix as the image changes.

Examples: regularized Gauss–Newton, Levenberg–Marquardt, and nonlinear Landweber methods.

Iteration can follow the nonlinear model more closely, but costs many forward solves and can depend strongly on the initial guess and model accuracy.

Constructive Direct Methods

The methods highlighted here share a foundation in *complex geometrical optics (CGO) solutions*: special solutions of the conductivity equation or related PDE with prescribed exponential behavior.

- **Calderón-type methods:** linearize about a constant background, estimate Fourier data for the conductivity perturbation, then apply Fourier inversion.
- **D-bar methods:** form nonlinear Fourier-like data using CGO solutions, use this as a parameter in a $\bar{\partial}$ equation which is solved to recover the conductivity.
- **VHPT:** transform CGO boundary traces into a virtual sinogram, then apply CT-style reconstruction.
- **Enclosure methods:** use CGO-based indicators to estimate supporting lines and recover the convex hull of conductivity inclusions.

Testing and Sampling Direct Methods

Rather than estimating conductivity values everywhere, these methods test candidate points or regions against the boundary data:

$$\text{candidate point or region} \xrightarrow{\text{boundary-data test}} \text{anomaly indicator.}$$

Examples include

- factorization methods,
- monotonicity methods,
- direct-sampling methods.

Typical Output

Repeating the test across the domain produces an indicator map or an estimate of the anomaly support.

Bayesian Reconstruction

Bayesian approaches are typically built on one-step or iterative model-based methods, but they represent uncertainty in the conductivity and in measurement or modeling errors using probability distributions.

They infer a probability distribution of plausible conductivities rather than only a single reconstructed image.

Stronger statistical support is assigned to candidate images that:

- Predict voltages consistent with the measurements and assumed error level.
- Are plausible under the prior assumptions, such as reasonable conductivity values or expected spatial structure.

What Bayesian Reconstruction Can and Can't Do

Bayesian inversion provides a flexible framework for incorporating many different kinds of prior knowledge and uncertainty.

We can obtain:

- A representative image, such as the most strongly supported image or an average of sampled images.
- Uncertainty information showing where plausible images agree or disagree.

But flexibility isn't free:

- Poor assumptions can bias the reconstruction. Very informative priors may overwhelm weak EIT data.
- Complicated priors can make computation extremely expensive.
- We still cannot recover information that the data fundamentally do not contain.

Finding one representative image may be manageable, but thoroughly characterizing the range of plausible images can require many forward solves, which may be very expensive.

Learning-Based and Hybrid Reconstruction

Neural networks can enter the reconstruction pipeline in several ways:

1. **End-to-end:** learn a direct map from EIT data to an image.
2. **Post-processing:** refine an initial classical reconstruction.
3. **Physics-guided or algorithm-unrolled:** incorporate the forward model, or turn several iterations of a reconstruction algorithm into trainable network stages.

Potential Benefits

- Very fast inference after training.
- Can exploit structure learned from training examples.

Central Risk

Performance may degrade when the anatomy or acquisition conditions differ from the training data.

Regularization Across Method Families

The shared goal is stability, but the mechanism depends on the method:

- **One-step and iterative model-based methods:** impose penalties or constraints; iterative methods may also stop early.
- **CGO-based constructive methods:** truncate or filter unstable transform data.
- **Testing and sampling methods:** replace exact operator tests with noise-tolerant versions, using spectral regularization or thresholds based on the noise level.
- **Bayesian and learned methods:** use explicit or learned priors to restrict plausible images.

The Shared Tradeoff

Stronger regularization improves stability, but may remove real detail or bias the reconstruction toward the assumed structure.

From Survey to Computation

EIT offers several routes from boundary data to interior information, but every route must control instability, noise, and modeling error.

We will now focus on a simple linearized one-step method, using the *complete electrode model (CEM)* for the forward model.

Later, D-bar reconstruction will provide a contrasting direct route from boundary data to an image.