Electrical Impedance Tomography: Computation and Applications

Part 3: Building a Mental Model of EIT

Melody Alsaker

Jyväskylä Summer School 2026

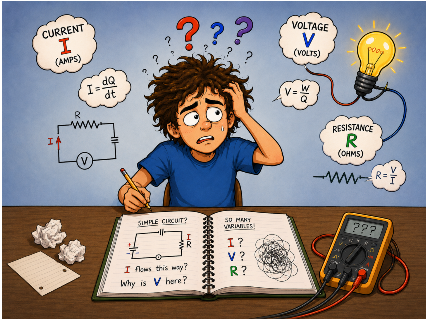
August 2026

Now that we've motivated the basics…

To understand *why* boundary voltages contain information about the interior, we need a mental model of how current flows.

Let's face it. . .

Unless you have background in electrical theory, quantities like *electrical current*, *voltage*, and *conductivity* are . . . not super intuitive.



For anyone here who isn't already an electricity expert, a hydraulic analogy will make things more concrete.

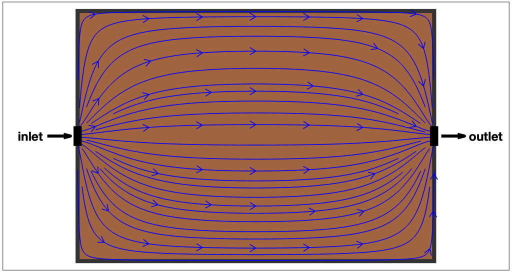
The Guiding Analogy

Instead of beginning with electricity, consider steady water flow through a shallow, rigid tank filled with a homogeneous, fully saturated porous material (e.g. sand).

We'll imagine the tank as a 2D object, as shown.

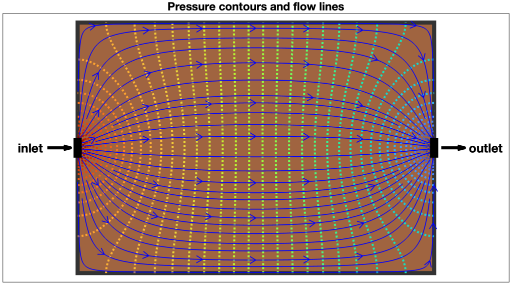
- Water is injected at some boundary locations and is removed at others.
- Since water is an incompressible fluid and the sand is fully saturated, the amount flowing in must equal the amount flowing out.

Flow lines



Pressure and Boundary Measurements

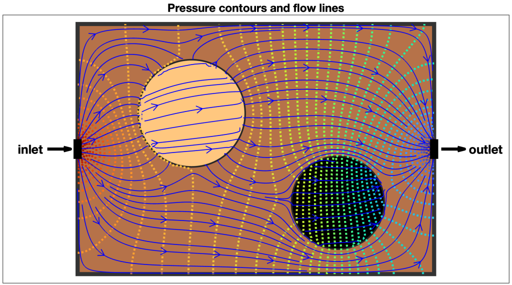
- The water flow induces a water pressure distribution inside the tank.
- Water always flows down the pressure gradient.
- Imagine that we place pressure sensors around the tank boundary.
- Each sensor would record a pressure value corresponding to the contour curve which intersects the boundary there.



Water pressure contours plotted as dashed lines

A Different Internal Distribution Changes the Data

- Now imagine that we insert inclusions of clay (black) and gravel (beige) into the tank.
- Water flows less easily through the clay and more easily through the gravel.
- This disrupts the fluid flow and also perturbs the pressure field.



Brown = sand, black = clay, beige = gravel

If we measured boundary pressures now, we'd get different readings than in the case of the homogeneous tank.

The fundamental question: Could we use the differences in those pressure measurements to figure out what's in the tank?

Let:

- $x =$ an arbitrary point in the tank
- $K(x) =$ *hydraulic conductivity*
  (a material property describing how easily water flows through media)
- $\mathbf{q}(x) =$ water flux (flow rate density)
- $p(x) =$ *hydraulic head*
  (a pressure-like potential that incorporates both water pressure and elevation)

Assuming our tank is level with the horizontal, differences in hydraulic head $p(x)$ are proportional to differences in water pressure, so we can think of this as the pressure field.

Then $-\nabla p$ points in the direction of decreasing pressure, which gives the direction of fluid flow.

Darcy's Law

Key Insight

Water flows down the gradient $-\nabla p$, but the *amount* of water flux $\mathbf{q}(x)$ at any point x also depends on the hydraulic conductivity $K(x)$ of the material.

In porous media, steady water flow may therefore be modeled by Darcy's law:

$$\mathbf{q} = -K\nabla p.$$

Conservation Gives the PDE

- We also know that water is never added or removed *inside* the tank (only on the boundary).
- Recall that the divergence $\nabla \cdot \mathbf{F}$ of a vector field $\mathbf{F}$ quantifies the pointwise net outflow per unit area of the vector field $\mathbf{F}$.
- So in our hydraulic scenario we have the conservation law

$$\nabla \cdot \mathbf{q} = 0$$

  (no flux sources or sinks) inside the domain of the tank.
- Combined with Darcy's law $\mathbf{q} = -K\nabla p$ we therefore have the steady-state Darcy flow equation

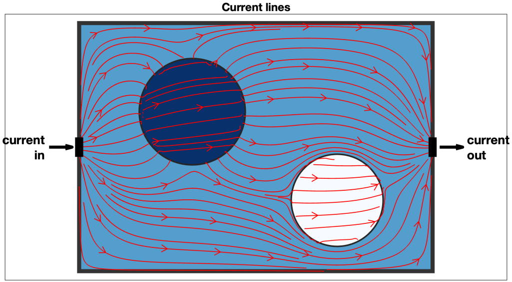
$$\nabla \cdot (K \nabla p) = 0$$

  inside the domain of the tank.

The EIT Version

Now, to understand EIT, we just change the physical interpretation of this model.

- Our tank is filled with salt water with inclusions of metal (dark blue) and silicon (white).
- *Electrical current* flows in through some boundary *electrodes*, and out through others.
- By conservation of charge, the total current entering through the boundary must equal the total current leaving.

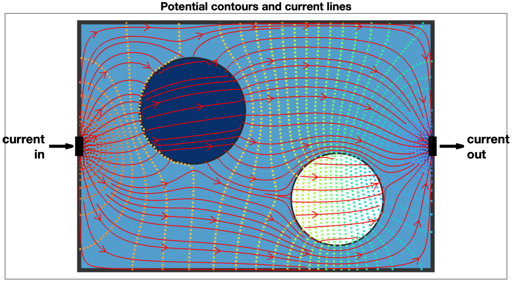


Light blue = salt water, white = silicon, dark blue = metal

Current flows easily through the metal (high electrical conductivity) but less easily through the silicon (lower conductivity).

Potential and Current Density

- The applied current induces an *electrical potential* field $u(x)$ inside the tank (like "electrical pressure").
- The current flows down the potential gradient $-\nabla u$, the same way water flows down $-\nabla p$.
- But the current density at any point also depends upon the electrical conductivity σ at that point.



Imagine that we place sensing electrodes around the tank that measure boundary electrical potential differences (i.e. boundary *voltages*).

Analogous Mathematical Formulations

Hydraulic Version

- $\mathbf{q}(x)$ = water flux (flow rate density)
- $p(x)$ = a pressure-like potential
- $K(x)$ = hydraulic conductivity
- Darcy's law: $\mathbf{q} = -K\nabla p$
- Conservation law: $\nabla \cdot \mathbf{q} = 0$
- Darcy flow equation:
  $\nabla \cdot (K\nabla p) = 0$

EIT Version

- $\mathbf{J}(x)$ = charge flux (current density)
- $u(x)$ = electrical potential
- $\sigma(x)$ = electrical conductivity
- Ohm's law: $\mathbf{J} = -\sigma\nabla u$
- Conservation law: $\nabla \cdot \mathbf{J} = 0$
- Conductivity equation:
  $\nabla \cdot (\sigma\nabla u) = 0$

The math is basically the same, only the physical interpretations are different!

The EIT Forward Problem: Intuition

Now we will focus only on the EIT version. We'll eventually make this more physically and mathematically precise, but this is a good starting mental model:

First, the *forward* problem.

Given the conductivity equation

$$\nabla \cdot (\sigma \nabla u) = 0$$

suppose we know the conductivity distribution $\sigma(x)$ everywhere in our target domain, along with the applied boundary current density.

This information allows us to solve for the potential $u(x)$ throughout the target and therefore predict the resulting boundary voltages.

This problem is readily numerically solvable using finite element methods.

The EIT Inverse Problem: Intuition

But in EIT, we don't know $\sigma(x)$ in advance... that's what we're trying to recover! So we wish to solve the *inverse* problem:

Given the conductivity equation

$$\nabla \cdot (\sigma \nabla u) = 0,$$

we know *both* of these boundary measurements:

- the applied boundary current density
- the measured boundary voltages (potential differences)

Can we use this boundary information to recover the internal conductivity $\sigma(x)$?

This problem is *much* harder, mathematically, than the forward problem!

The Mental Model

The Forward Problem

Given the conductivity and an applied current pattern,

$$\boxed{\sigma(x) + \text{applied current}} \longrightarrow \boxed{\text{boundary voltages}}.$$

The Inverse Problem

Repeat the experiment using many current patterns:

$$\boxed{\text{applied currents} + \text{measured voltages}} \longrightarrow \boxed{\text{estimate of } \sigma(x)}.$$

Internal conductivity σ cannot be observed directly, but it leaves a measurable signature in the boundary voltages.