Pair 2: AC Measurements and EIT Frames

Background: Read this first

In lecture we noted that EIT systems apply AC currents and use *synchronous demodulation* to isolate the voltage response at the drive frequency. Here we read a little background on this idea and then answer some questions using a simplified four-sample model.

For the n^{th} current-injection experiment, the time-dependent AC current pattern is $\mathbf{i}^{(n)}(t) = \mathbf{I}^{(n)}\cos(\omega t)$, where $\mathbf{I}^{(n)}$ is its constant amplitude vector and ω is the angular drive frequency. The quantity ωt is called the *phase*. In this simplified model, we sample the voltage signal at several phases and use demodulation to estimate the desired voltage component at the drive frequency.

Although a real voltage response may involve a phase shift from the applied current, in this problem we retain only the in-phase component, which is proportional to $\cos(\omega t)$. At electrode ℓ, we write this desired in-phase component as $v_\ell(t) = V_\ell \cos(\omega t)$, where V_ℓ is a signed coefficient. At four selected reference phases $\omega t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, the samples of $v_\ell(t)$ form the vector $V_\ell \mathbf{c}$, where $\mathbf{c}$ is the *reference cosine* vector

$$\mathbf{c} = \begin{bmatrix}\cos 0 & \cos\frac{\pi}{2} & \cos\pi & \cos\frac{3\pi}{2}\end{bmatrix}^\top = \begin{bmatrix}1 & 0 & -1 & 0\end{bmatrix}^\top.$$

The raw sample vector may also contain an unwanted contribution $\mathbf{r}_\ell$ due to measurement noise and other effects, so that we actually measure $\mathbf{y}_\ell = V_\ell\mathbf{c} + \mathbf{r}_\ell$. We estimate V_ℓ by computing the coefficient of the projection onto $\mathbf{c}$:

$$D(\mathbf{y}_\ell) = \frac{\mathbf{y}_\ell^\top \mathbf{c}}{\mathbf{c}^\top \mathbf{c}}.$$

The quantity $D(\mathbf{y}_\ell)$ is used as an estimate of V_ℓ and is the demodulated voltage measurement reported for electrode ℓ.

Choose exactly one: Problem A or Problem B.

Problem 2A (Hand Computations) [2 points]

(a) Verify that if $\mathbf{r}_\ell = \mathbf{0}$, then $D(\mathbf{y}_\ell) = V_\ell$. Briefly interpret this in terms of the physical problem.

(b) Verify that an unwanted component satisfying $\mathbf{r}_\ell^\top \mathbf{c} = 0$ makes no contribution to $D(\mathbf{y}_\ell)$, whereas a component of the form $\mathbf{r}_\ell = \epsilon\mathbf{c}$, where $\epsilon \in \mathbb{R}$, cannot be distinguished from the desired response. Briefly interpret this in terms of the physical problem.

(c) Consider an idealized EIT system with four electrodes. In the first adjacent-drive experiment, the current-amplitude vector is $\mathbf{I}^{(1)} = \begin{bmatrix}1 & -1 & 0 & 0\end{bmatrix}^\top$ mA, so the time-dependent applied current is $\mathbf{i}^{(1)}(t) = \mathbf{I}^{(1)}\cos(\omega t)$. The raw single-ended voltage readings are arranged in the matrix

$$Y^{(1)} = \begin{bmatrix} 2.2 & 0.2 & -1.8 & 0.2 \\ 0.9 & -0.1 & -1.1 & -0.1 \\ -0.7 & 0.3 & 1.3 & 0.3 \\ -2.0 & 0.0 & 2.0 & 0.0 \end{bmatrix} \text{ mV.}$$

Each row corresponds to one electrode, and the columns correspond, in order, to the four sampling phases above. Thus, $\mathbf{y}_\ell^\top$ is row ℓ of $Y^{(1)}$.

Compute the demodulated voltage measurement $D(\mathbf{y}_1)$ corresponding to electrode 1. Explain briefly why adding the same constant to all four samples from one electrode does not change the result.

Problem 2B (MATLAB) [2 points]

The following code stores raw voltage samples from four current-injection experiments in the array Y. The matrix `Y(:,:,n)` corresponds to experiment n, with electrodes indexed by rows and sampling phases by columns. The data include constant measurement offsets, representing DC baseline voltages from the electrode interfaces or measurement electronics that add approximately the same value to all four phase samples. Because these offsets are measurement artifacts rather than part of the voltage response at the drive frequency, we want the demodulation process to remove them.

```
Y(:,:,1) = [ 2.2  0.2 -1.8  0.2;
             0.9 -0.1 -1.1 -0.1;
            -0.7  0.3  1.3  0.3;
            -2.0  0.0  2.0  0.0];
Y(:,:,2) = [-1.4  0.1  1.6  0.1;
             2.3 -0.2 -2.7 -0.2;
             1.5  0.0 -1.5  0.0;
            -0.2  0.3  0.8  0.3];
Y(:,:,3) = [-1.45 -0.2  1.05 -0.2;
            -2.15  0.1  2.35  0.1;
             1.95  0.2 -1.55  0.2;
             0.65 -0.1 -0.85 -0.1];
Y(:,:,4) = [ 2.2  0.2 -1.8  0.2;
            -0.1 -0.1 -0.1 -0.1;
            -0.9  0.1  1.1  0.1;
             2.8 -0.2 -3.2 -0.2];
```

All voltage values are in mV.

(a) Construct the reference vector $\mathbf{c}$ in MATLAB. Using a loop and matrix-vector multiplication, demodulate the four experiments according to $\widehat{\mathbf{V}}^{(n)} = Y^{(n)}\mathbf{c}/(\mathbf{c}^\top\mathbf{c})$ and assemble the results as the columns of a demodulated voltage matrix `Vframe`. Display `Vframe`. Explain why the constant measurement offsets do not affect the result.

(b) Make each column of `Vframe` zero mean by subtracting its mean over the electrodes. Store the result as a final voltage matrix `Vzero` and display it. Explain why this changes the voltage reference but leaves all electrode-to-electrode voltage differences unchanged.