Return by tue 16.11. by 2 pm.

1. *Diamagnetic susceptibility of atomic hydrogen.* (See also Kittel problem 14.1) The wave function of the hydrogen atom in its ground state is $\psi = (\pi a_0^3)^{-1/2} \exp(-r/a_0)$, where $a_0 = \hbar^2/me^2$ is the Bohr radius. Show that for this state $\langle r^2 \rangle = 3a_0^2$ and calculate the molar diamagnetic (Larmor) susceptibility of atomic hydrogen. **(1 point)**

2. *Landau Levels.* (See also Kittel problem 9.11.) The Hamiltonian of a free electron without spin is

$$H = \frac{1}{2m}(-i\hbar\nabla + e\mathbf{A})^2.$$

The vector potential of a uniform magnetic field $B\mathbf{z}$ is $\mathbf{A} = -By\mathbf{x}$ in the Landau gauge. We will look for an eigenfunction of the Schrödinger equation $H\psi = \varepsilon\psi$ in the form

$$\psi(x, y, z) = \chi(y) \exp[i(k_x x + k_z z)].$$

Show that $\chi(y)$ satisfies the equation

$$-\frac{\hbar^2}{2m}\chi''(y) + 1/2m\omega_c^2(y - y_0)^2\chi(y) = (\varepsilon - \hbar^2 k_z^2/2m)\chi(y),$$

where $\omega_c = eB/m$ and $y_0 = -\hbar k_x/eB$. Argue that this equation yields the energies of the Landau levels,

$$\varepsilon_\nu = \hbar\omega_c(\nu + 1/2) + \frac{\hbar^2 k_z^2}{2m}, \qquad \nu = 0, 1, 2, \ldots$$

**(3 points)**

3. *Pauli spin susceptibility.* (See also Kittel problem 14.5) Let $n^+ = \frac{1}{2}n(1 + \xi)$ and $n^- = \frac{1}{2}n(1 - \xi)$ be the consentrations of spin-up and spin-down elctrons, respectively. Show that in a magnetic field $B$ the total energy of the spin-up electrons in a free-electron gas is

$$E^+ = E_0(1 + \xi)^{5/3} - \frac{1}{2}n\mu B(1 + \xi),$$

where $E_0 = \frac{3}{10}n\varepsilon_F$, in terms of the Fermi energy $\varepsilon_F$ in zero magnetic field. Find a similar expression for $E^-$. Minimise $E = E^+ + E^-$ with respect to $\xi$ and solve for the equilibrium value of $\xi$ in approximation $\xi \ll 1$. Go on to show that the magnetisation is $M = 3n\mu^2 B/2\varepsilon_F$, in agreement with Elliot equation (7.189) or Kittel equation (14.42). **(2 points)**