Return by tue 9.11. by 2 pm.

1. *Simple model for quantum dots* (See the lecture notes given by Kimmo) Show that the eigenenergies of a two-dimensional harmonic potential are $\varepsilon_l = \hbar\omega(l+1)$, and degeneracies $d_l = l+1$ with $l = 0, 1, 2, \ldots$ (Hint: Show that the Hamiltonian separates into a sum of two one-dimensional Hamiltonians and use the energies of one-dimensional harmonic oscillator.)

   In the capasitive model of a quantum dot the addition energy is

   $$\Delta_2(N) = \varepsilon_{N+1} - \varepsilon_N + e^2/C,$$

   where ε_N is the energy of the single-electron level occupied by the Nth electron and e^2/C is the capasitive charging energy. Use the simple model of a quantum dot where the non-interacting electrons occupy the levels of the two-dimensional harmonic oscillator, and plot the addition energies $\Delta_2(N)$ as a function of electron number N, for N=1…13. For simplicity use units where $\hbar\omega = e = 1$ and $C = N$, thus assume that the capasitance increases with the electron number. Compare your plot with the experimental one (see the lecture notes). What are the qualitative differences and similarities?**(2 points)**

2. Calculate the atomic polarizability, and hence the dielectric constant ϵ, of liquid Argon, for which the atomic number density is 2.128×10^{28} m^{-3} and the atomic radius is 1.18×10^{-10} m. Use the so called Clausius-Mossotti relation

   $$\frac{\epsilon - 1}{\epsilon + 2} = \frac{n\alpha}{3\epsilon_0}.$$

   (The experimental dielectric constant is 1.538) **(2 points)**

3. *Frequency-dependent atomic polarizability.* Allow the local field acting on an ion be frequency-dependent, $\mathbf{E}_{loc} = \mathbf{E_0}\exp(-i\omega t)$. The simplest classical theory of atomic polarizability treats the ion as an electronic shell of charge Ze and mass Zm tied to a rigid ion core by a harmonic spring, of spring constant $K = Zm\omega_0^2$. Argue that the equation of motion for the electronic shell is

   $$Zm\ddot{\mathbf{r}} = -K\mathbf{r} - Ze\mathbf{E}_{loc},$$

   where $\mathbf{r}$ is the displacement from equilibrium. By assuming that $\mathbf{r} = \mathbf{r}_0\exp(-i\omega t)$ show that the frequency-dependent atomic polarizability is

   $$\alpha(\omega) = \frac{Ze^2}{m(\omega_0^2 - \omega^2)}.$$

   (2 points)