Return by tue 26.10. by 2 pm.

1. Calculate the specific heat of a non-interacting electron gas at low temperatures and show that it increases linearly as a function of temperature. Hint: see for example Aschroft & Mermin: Solid state physics, pages 44-47 or Marder page 149.
Assume that the electrons in a superconductor form Cooper pairs which at $T = 0$ occupy the ground state ε_0. Assume further that the lowest excited state of a Cooper pair has energy $\varepsilon_0 + \Delta$. Show that the specific heat of such a two-state system depends exponentially on the temperature at low temperatures. Hint: Choose $\varepsilon_0 = 0$ and calculate the canonical partition function. **(3 points)**

2. *Magnetic field penetration in a superconducting plate.* Kittel problem 12.1. **(2 points)**

3. *Diffraction effect in Josephson junctions.* (See Marder problem 27.3 or Kittel problem 12.6) In the presence of a magnetic field the current density through a Josephson junction can be written as

$$\mathbf{j} = \mathbf{j}_0 \sin\left(\theta_2 - \theta_1 + \frac{2\pi}{\Phi_0}\int_{\gamma_{12}} \mathbf{A}\cdot \mathrm{d}\mathbf{l}\right), \tag{1}$$

where $\theta_2 - \theta_1$ is the phase difference between the superconductors, Φ_0 is the flux quantum and $\mathbf{A}$ is the vector potential of the magnetic field (compare with Elliot's eq. (6.183)). The path γ_{12} in the line integral joins the two superconductors. Show that the maximum zero-voltage current able to flow through a rectangular Josephson junction in the presence of a magnetic field is (the set-up is in the figure below)

$$\mathbf{J}_c = \mathbf{J}_0 \left|\frac{\sin(2\pi\Phi/\Phi_0)}{2\pi\Phi/\Phi_0}\right|. \tag{2}$$

1. Adopt Landau gauge $\mathbf{A} = (0, Bx, 0)$. Show that it yields $\mathbf{B} = (0, 0, B)$ and calculate the line integral across the junction.
2. Integrate the current density across the cross-section of the junction in order to get the total current.
3. Choose the phase difference $\theta_2 - \theta_1$ to maximize the zero-voltage current. **(2 points)**