Return by tue 30.11. by 2 pm.

1. *Ground-state energy of a simple antiferromagnet.* Show that the ground-state energy of the four spin antiferromagnetic nearest-neighbour Heisenberg linear chain,

$$\mathcal{H} = J(\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{S}_2 \cdot \mathbf{S}_3 + \mathbf{S}_3 \cdot \mathbf{S}_4 + \mathbf{S}_4 \cdot \mathbf{S}_1)$$

is

$$E_0 = -4JS^2\hbar^2 \left[1 + \frac{1}{2S}\right].$$

Furthermore derive all the eigenenergies of the Hamiltonian $\mathcal{H}$ in the case $S = 1/2$.
Hint: Write the Hamiltonian in the from $\mathcal{H} = \frac{1}{2}J[(\mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4)^2 - (\mathbf{S}_1 + \mathbf{S}_3)^2 - (\mathbf{S}_2 + \mathbf{S}_4)^2]$, thus use the addition of angular momenta. **(3 points)**

2. Determine the spectroscopic term symbol for the ground-state of the elements Sc, Ti, V, Cr, Mn, Fe, Co and Ni. (Write also down the steps in applying the Hund rules, not just the plain result) **(2 points)**

3. *Magnon dispersion relation.* Read the section 7.2.5.4 (pages 620-624) *Spin waves: magnons* from Elliot and derive the magnon dispersion relation

$$\hbar\omega_k = 2JS(1 - \cos(ka)).$$

(3 points)