MATERIALS PHYSICS II FALL 2007 HOMEWORK PROBLEMS 9

Return to Robert (mailbox Physics 2nd floor) by Tuesday 20.11.2007 at noon.

Comment: working through the weekly problems as team work is completely acceptable and even recommended, as far as each person writes down and returns his/her own solutions!

1. The propagator of a one-particle system with Hamiltonian $\hat{h}$

$$\hat{h} = -\frac{\hbar^2}{2m}\nabla^2 + v(\mathbf{r})$$

and with stationary eigenstates $\varphi_n(\mathbf{r})$ and eigenenergies ϵ_n is defined as

$$K(\mathbf{r}_1, \mathbf{r}_2; t_1 - t_2) = \sum_n^\infty e^{-\frac{i\epsilon_n}{\hbar}(t_1 - t_2)}\varphi_n(\mathbf{r}_1)\varphi^*(\mathbf{r}_2)$$

a) Let $\Psi_0(\mathbf{r})$ be an arbitrary wave function. Show that

$$\Psi(\mathbf{r}, t) = \int d\mathbf{r}' K(\mathbf{r}, \mathbf{r}'; t - t_0)\Psi_0(\mathbf{r}') \tag{1}$$

solves the time-dependent Schródinger equation and satisfies the initial condition $\Psi(\mathbf{r}, t_0) = \Psi_0(\mathbf{r})$.

b) Let $\Psi_0(\mathbf{r}) = \delta(\mathbf{r} - \mathbf{r}_0)$ be a delta function at point $\mathbf{r}_0$. If we insert this into Eq.(1) what does this tell you about the physical interpretation of the propagator?

c) Consider now a free particle in one dimension such that the eigenstates are given by

$$\phi_k(x) = \frac{1}{\sqrt{L}}e^{ikx}$$

where L is the volume of the system (we use the usual periodic boundary conditions). Calculate the propagator K.

d) Let the initial wave function be given as a normalized Gaussian

$$\Psi_0(x) = (\pi a^2)^{-\frac{1}{4}} \exp\left(\frac{i}{\hbar}p_0 x - \frac{x^2}{2a^2}\right)$$

Find the solution to the time-dependent Schrödinger equation.Interpret the solution.

To answer these questions you may find the following formula useful

$$\int_{-\infty}^{\infty} dx e^{-ax^2 - bx} = \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{4a}} \tag{2}$$

(5 points)

2. Read the attached article (Foley, Candela, Martini, Tuominen, AJP 67, 389 (1999)) and answer briefly the following questions. You may find it useful to look at a review article by Agrait et al., linked on the course web-page (note: this file is password-protected, the password is "fysm400").
(a) The conductance $G = I/V$ through a nanometer-sized wire is experimentally found to be *quantized*, ie., it has discrete values in multiples of $G_0 = 2e^2/h$. Derive this result treating the wire to act as a wave-guide, transmitting electron "waves" under certain conditions.
(b) Describe the experimental setup.
(c) Interpret figure 2. **(5 points)**