MATERIALS PHYSICS II FALL 2007 HOMEWORK PROBLEMS 8

Return to Robert (mailbox Physics 2nd floor) by Tuesday 13.11.2007 at noon.
Comment: working through the weekly problems as team work is completely acceptable and even recommended, as far as each person writes down and returns his/her own solutions!

1. (a) Assume the relaxation time in copper to be 20×10^{-14} s. How strong an electric field would be needed in order to have one Bloch oscillation in less than the relaxation time?
(b) Suppose that it is possible to apply an electric field with such a strength that the answer to (a) indicates. If the electrons would produce a current with the Drude conductivity, how much power would be dissipated per volume unit, and how much the metal would heat up ?
(c) In GaAs, the relaxation time can be as long as 3×10^{-10} s and one can fabricate artificial layer structures where the supercell is of the order of 100 Angstroms. How strong electric field is now needed to produce Bloch oscillations? **(4 points)**

2. Consider a 2D tight-binding model with energy dispersion

$$\epsilon(\mathbf{k}) \propto 2s[cos(k_x a) + cos(k_y a)].$$

Write the effective mass tensor. How does the effective mass depend on s, and why?
(2 points)

3. Show that if one begins with a true Bloch state and subjects it to a weak electric field, it evolves into a new Bloch state with the wave vector obeying

$$\hbar\dot{\mathbf{k}} = -e\mathbf{E}.$$

Do as follows:
(a) Take a Bloch state and evolve it by a small amount dt forward in time, using the Hamiltonian

$$\hat{H} = \frac{\hat{P}^2}{2m} + U(\hat{R}) + e\mathbf{E} \cdot \hat{R}.$$

(b) Apply the translation operator $T_R^\dagger$. Interpret the result to show that a new Bloch state with a new k-vector is obtained, to first order in dt. What is the new k-vector $\mathbf{k}(dt)$? **(4 points)**