MATERIALS PHYSICS II FALL 2007 HOMEWORK PROBLEMS 6

Return to Robert (mailbox Physics 2nd floor) by Tuesday 23.10.2007 at noon.
Comment: working through the weekly problems as team work is completely acceptable and even recommended, as far as each person writes down and returns his/her own solutions!

1. Consider a one-dimensional system of harmonic oscillators with the zero-point energy of 10 meV. At what temperature the occupation of the first excited state is 50% of the occupation of the ground state? What is then the heat capacity? Compare the result to the classical limit for C_V. **(2 points)**

2. The attached figure (turn page) shows the heat capacity C_V of a system as a function of temperature. Show that the shaded area in the figure corresponds to the quantum mechanical zero-point energy. **(2 points)**

3. Estimate the speeds of sound (longitudinal and transverse, in m/s) in silicon, based on the dispersion curve shown in Marder's figure 13.6, shown also in the lecture notes. The lattice parameter of silicon is 5.43 Angstrom. **(2 points)**

4. Assume that the system has only two available energy states, the ground state ϵ_0 and one excited state $\epsilon_0 + \Delta$. Show that the specific heat depends exponentially on temperature, when $k_B T << \Delta$:

$$c_V \approx k_B \left(\frac{\Delta}{k_B T}\right)^2 e^{-\Delta/k_B T}.$$

(2 points)

5. Consider the Hamiltonian of a single harmonic oscillator:

$$\hat{H} = \frac{\hat{P}^2}{2M} + \frac{1}{2} M\omega^2 \hat{R}^2.$$

(a) Express $\hat{P}$ in terms of lowering and raising operators $\hat{a}$, $\hat{a}^\dagger$, respectively. These operators are defined in the lecture notes and also in Marder's equation (13.33).
(b) Express $\hat{R}$ in terms of $\hat{a}$, $\hat{a}^\dagger$.
(c) Show that $[\hat{a}, \hat{a}^\dagger] = 1$
(d) Show that the Hamiltonian can be written as $\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})$. **(2 points)**