MATERIALS PHYSICS II FALL 2007 HOMEWORK PROBLEMS 11

Return to Robert (mailbox Physics 2nd floor) by Tuesday 11.12.2007 at noon.

Comment: working through the weekly problems as team work is completely acceptable and even recommended, as far as each person writes down and returns his/her own solutions!

1. De Haas - van Alphen oscillations of the magnetization in Cu, corresponding to the "belly orbit", have a period in (1/B) of 1.83 $\times 10^{-5}$ T^{-1}. Obtain a value for the cross-sectional area of the Fermi surface, and compare your answer with the value for the free-electron Fermi sphere (the atomic density of Cu is 8.45 $\times 10^{28}$ m^{-3}).

(2 points)

2. *Curie's law of paramagnetism.* (See also Ashcroft & Mermin: Solid State Physics problem 31.9) The partition function (Elliot eq. (7.167)) can be written as

$$Z = e^{-\beta F} = \sum_m e^{-\beta E_m} = \sum_{m=-j}^{j} \langle jm|e^{-\beta H}|jm\rangle = \mathrm{Tr}\{e^{-\beta H}\}$$

where $H = g_J \mu \mathbf{H} \cdot \mathbf{J}$ is the Hamiltonian with magnetic field $\mathbf{H}$ and angular momentum operator $\mathbf{J}$ and F is the Helmholtz free energy. The trace is carried out in the eigenbasis of J^2. Furthermore $\mu = \mu_0 \mu_B$ and $\beta = 1/k_B T$. The susceptibility is then defined as

$$\chi = -\frac{n}{\mu_0}\frac{\partial^2 F}{\partial H^2}$$

for a concentration n of atomic moments. The Curie's law can be deduced at high temperature without going through the algebra of Brillouin functions, for we can expand $\exp(-\beta H) = 1 - \beta H + (\beta H)^2/2 - \ldots$. Evaluate the free energy to second order in the field, using the fact that for the cartesian components of angular momentum operator (J_x, J_y, J_z) we have

$$\mathrm{Tr}\{J_i J_j\} = \frac{1}{3}\delta_{ij}\mathrm{Tr}\{J^2\}$$

and extract the high-temperature ($g_J \mu H \ll k_B T$) susceptibility (Elliot eq. (7.174)). **(4 points)**

3. *Ground-state energy of a simple antiferromagnet.* Show that the ground-state energy of the four spin antiferromagnetic nearest-neighbour Heisenberg linear chain,

$$H = J(\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{S}_2 \cdot \mathbf{S}_3 + \mathbf{S}_3 \cdot \mathbf{S}_4 + \mathbf{S}_4 \cdot \mathbf{S}_1)$$

is

$$E_0 = -4JS^2\hbar^2\left[1 + \frac{1}{2S}\right].$$

Furthermore derive all the eigenenergies of the Hamiltonian H in the case $S = 1/2$.
Hint: Write the Hamiltonian in the from $H = \frac{1}{2}J[(\mathbf{S}_1+\mathbf{S}_2+\mathbf{S}_3+\mathbf{S}_4)^2-(\mathbf{S}_1+\mathbf{S}_3)^2-(\mathbf{S}_2+\mathbf{S}_4)^2]$, thus use the addition of angular momenta. (**4 points**)