

Return to Michael by Wednesday 15.11.2006 at noon

1. Curie's law of paramagnetism. (See also Ashcroft & Mermin: Solid State Physics problem 31.9) The partition function (Elliot eq. (7.167)) can be written as

$$Z = e^{-\beta F} = \sum_m e^{-\beta E_m} = \sum_{m=-j}^j \langle jm | e^{-\beta H} | jm \rangle = \text{Tr}\{e^{-\beta H}\}$$

where $H = g_J \mu \mathbf{H} \cdot \mathbf{J}$ is the Hamiltonian with magnetic field \mathbf{H} and angular momentum operator \mathbf{J} and F is the Helmholtz free energy. The trace is carried out in the eigenbasis of J^2 . Furthermore $\mu = \mu_0 \mu_B$ and $\beta = 1/k_B T$. The susceptibility is then defined as

$$\chi = -\frac{n}{\mu_0} \frac{\partial^2 F}{\partial H^2}$$

for a concentration n of atomic moments. The Curie's law can be deduced at high temperature without going through the algebra of Brillouin functions, for we can expand $\exp(-\beta H) = 1 - \beta H + (\beta H)^2/2 - \dots$. Evaluate the free energy to second order in the field, using the fact that for the cartesian components of angular momentum operator (J_x, J_y, J_z) we have

$$\text{Tr}\{J_i J_j\} = \frac{1}{3} \delta_{ij} \text{Tr}\{J^2\}$$

and extract the high-temperature ($g_J \mu H \ll k_B T$) susceptibility (Elliot eq. (7.174)). **(4 points)**

2. Ground-state energy of a simple antiferromagnet. Show that the ground-state energy of the four spin antiferromagnetic nearest-neighbour Heisenberg linear chain,

$$H = J(\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{S}_2 \cdot \mathbf{S}_3 + \mathbf{S}_3 \cdot \mathbf{S}_4 + \mathbf{S}_4 \cdot \mathbf{S}_1)$$

is

$$E_0 = -4JS^2\hbar^2 \left[1 + \frac{1}{2S} \right].$$

Furthermore derive all the eigenenergies of the Hamiltonian H in the case $S = 1/2$.

Hint: Write the Hamiltonian in the form $H = \frac{1}{2}J[(\mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4)^2 - (\mathbf{S}_1 + \mathbf{S}_3)^2 - (\mathbf{S}_2 + \mathbf{S}_4)^2]$, thus use the addition of angular momenta. **(3 points)**

3. Determine the spectroscopic term symbol for the ground-state of the elements Sc, Ti, V, Cr, Mn, Fe, Co and Ni. (Write also down the steps in applying the Hund rules, not just the plain result) **(3 points)**