MATERIALS PHYSICS II FALL 2006 HOMEWORK PROBLEMS 7

Return to Michael by Wednesday 8.11.2006 at noon

1. *Pauli spin susceptibility.* (See also Kittel problem 14.5) Let $n^+ = \frac{1}{2}n(1+\xi)$ and $n^- = \frac{1}{2}n(1-\xi)$ be the consentrations of spin-up and spin-down elctrons, respectively. Show that in a magnetic field B the total energy of the spin-up electrons in a free-electron gas is

$$E^+ = E_0(1+\xi)^{5/3} - \frac{1}{2}n\mu B(1+\xi),$$

where $E_0 = \frac{3}{10}n\varepsilon_F$, in terms of the Fermi energy ε_F in zero magnetic field. Find a similar expression for E^-. Minimise $E = E^+ + E^-$ with respect to ξ and solve for the equilibrium value of ξ in approximation $\xi \ll 1$. Go on to show that the magnetisation is $M = 3n\mu^2 B/2\varepsilon_F$, in agreement with Elliot equation (7.189) or Kittel equation (14.42). **(3 points)**

2. Consider an alternative way of deriving the (Larmor) atomic diamagnetic susceptibility. Do the following steps: (i) Assume $\mathbf{B}_{ext}$ parallel to z-axis, take the vector potential $\mathbf{A} = -\frac{1}{2}(\mathbf{r} \times \mathbf{B}_{ext})$ and develop the kinetic part of the Hamiltonian, $H_{kin} = \frac{1}{2m_e}\sum_i(\mathbf{p} + e\mathbf{A})^2$ where the sum is over all the electrons in the atom. (ii) Evaluate the expectation value of the atomic magnetic moment in a state $|\phi>$:

$$\mu_m = -\frac{\partial}{\partial \mathbf{B}_{ext,z}} < \phi|H|\phi >$$

and separate from the result two parts, the first one proportional to $< \phi|\mathbf{L}_z|\phi >$ and the second one to $< \phi|\sum_i(x_i^2 + y_i^2)|\phi >$. From the second term, derive the result

$$\chi_{m,dia} = -\frac{n_{at}}{6m_e}e^2\mu_0 \sum_i < \phi|r_i^2|\phi > .$$

Compare the result to the one derived in the lecture: $\chi_{m,dia} = -n_{at}Ze^2\mu_0 < r^2 > /6m_e$. (Hint: see Elliott page 576.) (**3 points**)

3. De Haas - van Alphen oscillations of the magnetization in Cu, corresponding to the "belly orbit", have a period in (1/B) of 1.83 $\times 10^{-5}$ T^{-1}. Obtain a value for the cross-sectional area of the Fermi surface, and compare your answer with the value for the free-electron Fermi sphere (the atomic density of Cu is 8.45 $\times 10^{28}$ m^{-3}). (**3 points**)