MATERIALS PHYSICS II   FALL 2006   HOMEWORK PROBLEMS 6

Return to Michael by Wednesday 1.11.2006 at noon

1. *Diamagnetic susceptibility of atomic hydrogen.* The wave function of the hydrogen *1s* electron is $\psi = (\pi a_0^3)^{-1/2} \exp(-r/a_0)$, where $a_0 = \hbar^2/me^2$ is the Bohr radius. Show that for this state $\langle r^2 \rangle = 3a_0^2$ and calculate the molar diamagnetic (Larmor) susceptibility of atomic hydrogen. **(3 points)**

2. *Landau Levels.* The Hamiltonian of a free electron without spin is

$$H = \frac{1}{2m}(-i\hbar\nabla + e\mathbf{A})^2.$$

The vector potential of a uniform magnetic field $B\mathbf{z}$ is $\mathbf{A} = -By\mathbf{x}$ in the Landau gauge. We will look for an eigenfunction of the Schrödinger equation $H\psi = \varepsilon\psi$ in the form

$$\psi(x, y, z) = \chi(y) \exp[i(k_x x + k_z z)].$$

Show that $\chi(y)$ satisfies the equation

$$-\frac{\hbar^2}{2m}\chi''(y) + 1/2m\omega_c^2(y - y_0)^2\chi(y) = (\varepsilon - \hbar^2 k_z^2/2m)\chi(y),$$

where $\omega_c = eB/m$ and $y_0 = -\hbar k_x/eB$. Argue that this equation yields the energies of the Landau levels,

$$\varepsilon_\nu = \hbar\omega_c(\nu + 1/2) + \frac{\hbar^2 k_z^2}{2m}, \qquad \nu = 0, 1, 2, \ldots$$

(**4 points**)