MATERIALS PHYSICS II FALL 2006 HOMEWORK PROBLEMS 4

Return to Michael by Wednesday 11.10.2006 at noon

1. The classical electrostatic energy of an electron density distribution $n(\mathbf{r})$ is

$$E_{es}[n(\mathbf{r})] = \frac{e^2}{2}\int d\mathbf{r}\int d\mathbf{r}'\frac{n(\mathbf{r})n(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}.$$

Evaluate the functional derivative $\delta E_{es}/\delta n$. (**2 points**)

2. Consider for simplicity a system of only two electrons. Approximate the total wave function $\Psi(\mathbf{r}_1 s_1, \mathbf{r}_2 s_2)$ with a Slater determinant. Derive the expectation value of the Hamiltonian, $E =< \Psi|H|\Psi >$, where $H = h_1(\mathbf{r}_1, s_1) + h_2(\mathbf{r}_2, s_2) + v(|\mathbf{r}_1 - \mathbf{r}_2|)$, apply the variational principle with Lagrange multipliers ϵ and recover Hartree-Fock equations. (**4 points**)

3. Read through the material about Wigner crystals and "molecules" given on Wednesday's (4.10.) lecture and answer briefly (max. a half page / question):
(a) Describe the two melting mechanisms that are observed.
(b) Discuss the trends in the corresponding melting temperatures.
(c) What is a magic electron cluster? (**3 points**)