

Return to Michael by **Friday 8.12.2006** at noon! The discussion is **Monday 11.12** at 10:15 in FYS2

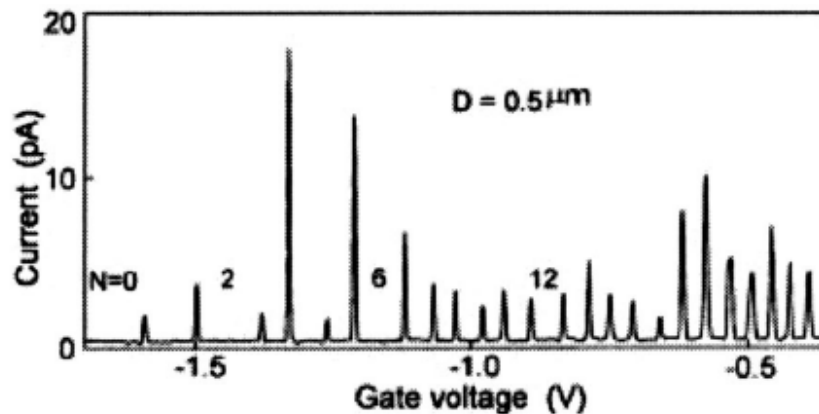
1. *Box potential model* Assume a box of constant volume and dimensions  $(a, a, b)$ . The potential inside of the box is zero and infinite outside. Find the ratios  $a/b$  that minimize the energy if you pack two or three electrons into this box. Which configuration do you expect a  $\text{Na}_3$  cluster to adopt according to this model (Na has one valence electron) ? **(3 points)**

2. *Simple model for quantum dots.* Show that the eigenenergies of a two-dimensional harmonic potential are  $\varepsilon_l = \hbar\omega(l + 1)$ , and degeneracies  $d_l = l + 1$  with  $l = 0, 1, 2, \dots$  (Hint: Show that the Hamiltonian separates into a sum of two one-dimensional Hamiltonians and use the energies of one-dimensional harmonic oscillator.)

In the capacitive model of a quantum dot the addition energy is

$$\Delta_2(N) = \varepsilon_{N+1} - \varepsilon_N + e^2/C,$$

where  $\varepsilon_N$  is the energy of the single-electron level occupied by the  $N$ th electron and  $e^2/C$  is the capacitive charging energy. Use the simple model of a quantum dot where the non-interacting electrons occupy the levels of the two-dimensional harmonic oscillator, and plot the addition energies  $\Delta_2(N)$  as a function of electron number  $N$ , for  $N=1 \dots 13$ . For simplicity use units where  $\hbar\omega = e = 1$  and  $C = N$ , thus assume that the capacitance increases with the electron number. Compare your plot with the experimental one (from Tarucha *et al.* Phys. Rev. Lett. **77** (1996) 3613):



What are the qualitative differences and similarities?  
**(3 points)**