

*Return to Michael by Wednesday 29.11.2006 at noon*

**1.** Calculate the specific heat of a non-interacting electron gas at low temperatures and show that it increases linearly as a function of temperature. Hint: see for example Ascroft & Mermin: Solid state physics, pages 44-47 or Marder page 149.

Assume that the electrons in a superconductor form Cooper pairs which at  $T = 0$  occupy the ground state  $\varepsilon_0$ . Assume further that the lowest excited state of a Cooper pair has energy  $\varepsilon_0 + \Delta$ . Show that the specific heat of such a two-state system depends exponentially on the temperature at low temperatures. Hint: Choose  $\varepsilon_0 = 0$  and calculate the canonical partition function. **(3 points)**

**2.** *Magnetic field penetration in a superconducting plate.* Kittel problem 12.1. **(3 points)**

**3.** *Diffraction effect in Josephson junctions.* (See Marder problem 27.3 or Kittel problem 12.6) In the presence of a magnetic field the current density through a Josephson junction can be written as

$$J = J_0 \sin \left( \theta_2 - \theta_1 + \frac{2\pi}{\Phi_0} \int_{\gamma_{12}} \mathbf{A} \cdot d\mathbf{l} \right), \quad (1)$$

where  $\theta_2 - \theta_1$  is the phase difference between the superconductors,  $\Phi_0$  is the flux quantum and  $\mathbf{A}$  is the vector potential of the magnetic field (compare with Elliot's eq. (6.183)). The path  $\gamma_{12}$  in the line integral joins the two superconductors. Show that the maximum zero-voltage current able to flow through a rectangular Josephson junction in the presence of a magnetic field is (for the set-up see Marder figure 27.5a)

$$\mathbf{J}_c = \mathbf{J}_0 \left| \frac{\sin(2\pi\Phi/\Phi_0)}{2\pi\Phi/\Phi_0} \right|. \quad (2)$$

1. Adopt Landau gauge  $\mathbf{A} = (0, Bx, 0)$ . Show that it yields  $\mathbf{B} = (0, 0, B)$  and calculate the line integral across the junction.
2. Integrate the current density across the cross-section of the junction in order to get the total current.
3. Choose the phase difference  $\theta_2 - \theta_1$  to maximize the zero-voltage current.

**(3 points)**