MATERIALS PHYSICS II FALL 2006 HOMEWORK PROBLEMS 1

Return to Michael by Tuesday 19.9.2006 at noon

1. *The Sommerfeld expansion.* The Sommerfeld expansion helps to evaluate integrals that are of the form

$$\int_{-\infty}^{\infty} H(\epsilon) f(\epsilon) d\epsilon \tag{1}$$

and meet two conditions: (i) f is the Fermi function $f(\epsilon) = [exp((\epsilon - \mu)/k_B T) + 1]^{-1}$ and (ii) $H(\epsilon)$ vanishes at the lower limit of the integral and does not diverge faster than ϵ^k at the upper limit. Show that under these conditions, in the neighborhood of $\epsilon = \mu$, the integral (1) can be evaluated as

$$\int_{-\infty}^{\infty} H(\epsilon) f(\epsilon) d\epsilon = \int_{-\infty}^{\mu} H(\epsilon) d\epsilon + \sum_{n=1}^{\infty} (k_B T)^{2n} a_n \frac{d^{2n-1}}{d\epsilon^{2n-1}} H(\epsilon)|_{\epsilon=\mu}$$

where a_n:s are dimensionless constants

$$a_n = \int_{-\infty}^{\infty} \frac{x^{2n}}{(2n)!} (-\frac{d}{dx} \frac{1}{e^x + 1}) dx \quad ; \quad x = (\epsilon - \mu)/k_B T$$

Hint: See e.g., Marder p 146 or AM Appendix C. (**4 points**)

2. *The free electron gas in two dimensions, AM problem 2.1.* For 3D electron gas, the following equations hold (n is density, k_F Fermi wave vector and r_S density parameter): $n = k_F^3/3\pi^2$ and $k_F = (9\pi/4)^{1/3}/r_S$.
(a) Derive the corresponding relations for two-dimensional electron gas.
(b) Show that for 2D electron gas, the density of electron levels $g(\epsilon)$ is an energy-independent constant, when $\epsilon > 0$ and zero when $\epsilon < 0$.
(c) Show that because $g(\epsilon)$ is constant, every term in the Sommerfeld expansion for n vanishes except the $T = 0$ term. Deduce that $\mu = \epsilon_F$ at any temperature. (**3 points**)

3. He^3 is a fermion with spin 1/2. The density of He^3 liquid is 0.081 gcm^{-3} at temperatures close to absolute zero. What is the Fermi energy and Fermi temperature of He^3 liquid? **(2 points)**

4. Consider a square lattice in 2D with a lattice parameter a and m electrons per unit cell. Plot the first four Brillouin zones (BZ). If $m = 3$, up to which BZ does k_F extend? **(2 points)**