Osittaisdifferentiaaliyhtälöt
DEMO 9

1.Write down an explicit formula for a solution of

$$\begin{cases} u_t - \Delta u + cu = f & \text{in } \mathbb{R}^n \times (0, \infty); \\ u = g & \text{on } \mathbb{R}^n \times \{t = 0\}, \end{cases}$$

where $c \in \mathbb{R}$.

2. Let $\phi : \mathbb{R} \to \mathbb{R}$ be smooth and convex. Assume that u solves the heat equation (that is, $u_t - \Delta u = 0$) and $v = \phi(u)$. Prove that v is a subsolution of the heat equation, i.e. $v_t - \Delta v \leq 0$.

3. Prove that $v = |Du|^2 + u_t^2$ is a subsolution of the heat equation, where u solves the heat equation.

4. Let u be a smooth solution of the following initial-boundary problem

$$\begin{cases} u_t - \Delta u = u & \text{in } \Omega_T = \Omega \times (0, T); \\ u = g & \text{on } \Gamma_T = (\Omega \times \{t = 0\}) \cup (\partial\Omega \times [0, T]), \end{cases}$$

where Ω is a bounded smooth domain, $T > 0$, and g is a continuous function. Prove that

$$|u(x, t)| \leq e^t \max_{\Gamma_T} |g|, \quad \forall\, (x, t) \in \Omega_T.$$