

Osittaisdifferentiaaliyhtälöt
DEMO 10

1. Solve the problem

$$\begin{cases} u_{tt} - u_{xx} = x^2 & \text{in } \mathbb{R} \times (0, \infty); \\ u(x, 0) = x; \\ u_t(x, 0) = 0. \end{cases}$$

2. Solve the problem

$$\begin{cases} u_{tt} - u_{xx} = xt & \text{in } \mathbb{R} \times (0, \infty); \\ u(x, 0) = 0; \\ u_t(x, 0) = 0. \end{cases}$$

3. Solve the problem

$$\begin{cases} u_{tt} - \Delta u = 0 & \text{in } \mathbb{R}^3 \times (0, \infty); \\ u(x, 0) = |x|^2; \\ u_t(x, 0) = x_3, & x = (x_1, x_2, x_3) \in \mathbb{R}^3. \end{cases}$$

4. Find an explicit formula for a solution of

$$\begin{cases} u_{tt} - u_{xx} = 0 & \text{in } \mathbb{R} \times (0, \infty); \\ u(0, t) = 0, & t > 0; \\ u(x, 0) = g(x), & x > 0; \\ u_t(x, 0) = h(x), & x > 0. \end{cases}$$

5. Solve the problem

$$\begin{cases} u_{tt} - \Delta u = e^{-t} & \text{in } \mathbb{R}^3 \times (0, \infty); \\ u(x, 0) = x_1; \\ u_t(x, 0) = x_2 x_3, & x = (x_1, x_2, x_3) \in \mathbb{R}^3. \end{cases}$$