Radio Resource Allocation for Collaborative OFDMA Relay Networks with Imperfect Channel State Information

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***Abstract*—This paper addresses the resource allocation problem in collaborative relay-assisted OFDMA networks. Recent works on the subject usually ignored either the selection of relays, asymmetry of the source-to-relay and relay-to-destination links or the imperfections of channel state information. In this article we take into account all these together and our focus is two-fold. Firstly, we consider the problem of asymmetric radio resource allocation, where the objective is to maximize the system throughput of the source-to-destination link under various constraints. In particular, we consider optimization of the set of collaborative relays and link asymmetries together with subcarrier and power allocation. Using a dual approach, we solve each sub-problem in an asymptotically optimal and alternating manner. Secondly, we pay attention to the effects of imperfections in the channel-state information needed in resource allocation decisions. We derive theoretical expressions for the solutions and illustrate them through simulations. The results validate clearly the additional performance gains through an asymmetric cooperative scheme compared to the other recently proposed resource allocation schemes.**

***Index Terms*—OFDMA, relay selection, subcarrier allocation, power allocation, imperfect CSI, cooperative communications.**

I. Introduction

Orthogonal Frequency Division Multiple Access (OFDMA) is an effective technique that exploits the features of Orthogonal Frequency Division Multiplexing (OFDM) in combating channel noise and multipath effects, and finally enables high data rate transmissions over fading channels. In addition, OFDMA is able to provide good bandwidth scalability as the number of subcarriers can be flexibly configured [1]. Therefore, OFDMA is widely adopted in many standards of existing/upcoming wireless communication systems, such as IEEE 802.11ac [2], LTE/LTE-A [3] and WiMAX [4].

Meanwhile, cooperative communication has emerged as one of the main trends to reach even better system performance in terms of throughput, energy efficiency or cell coverage. Therefore, the incorporation of OFDMA and cooperative relays is foreseen to result in a promising structure that offers the possibility to reach many desirable objectives for future wireless networks [5]. However, combination of a conventional one-to-many (single hop) OFDMA system and a relay network calls for a careful design of the radio resource allocation (RRA) principles. This means careful design and coordination of the power and subcarrier allocation, selection of relay(s) across different hops and optimizing the resource asymmetries between the hops.

The RRA algorithm plays an important role in either conventional or relay-aided OFDMA system [1]. Related works on the subject (see e.g. [6]-[12]) mostly assumed perfect channel state information (CSI) to be available at the source. An iterative algorithm was proposed in [6] to solve the subcarrier assignment together with relay selection. Subsequently, the power allocation problem was solved by another iterative method based on water-filling algorithm. Similar to [6], in [7] the optimization scheme was divided into two sub-problems without considering the relay selection. Two iterative methods were used with high computational complexity to solve the two sub-problems, respectively. The authors in [8] introduced closed-form solution for radio resource allocation for a multi-hop cooperative relay network. However, the per-tone power constraint was used. In [10] a threshold method was used to solve two sub-problems, which were subcarrier allocation and power allocation. Although the performance was improved compared to some other algorithms, the total power constraint was considered. This is not a realistic setting since each node has its own power limitation. The work in [11] also proposed a subcarrier and relay pairing algorithm to solve the existing RRA problem, which resulted in a method with high computational complexity. Moreover, all the previous works assumed the transmission durations for the base station and relay links to be equal, which can result in a reduction of degree of freedom and system throughput [12]. In [12], a study on the asymmetric resource allocation was presented. However, this work only considers single relay in the OFDMA networks without exploring cooperative diversity. Similarly, [13] also takes asymmetric link into consideration in the context of multi-relay networks without exploring relay selection advantages. Meanwhile, since a perfect CSI cannot be obtained by the source in practical work, a RRA scheme with imperfect CSI calls for careful research. [14] considers the RRA algorithm for conventional OFDMA networks without

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relays. A recent work in this line [15] investigated the issue of joint RRA and relay selection with imperfect CSI. The authors, however, focused on on power minimization and mean rate to characterize the CSI uncertainty, which results in different interpretations for system optimization. Another recent work about RRA for OFDMA relay networks with imperfect CSI was introduced in [16], where only one relay was selected for assisting the transmission.

The essence of this paper is to consider all the above-mentioned fundamental properties jointly: the selection of the relay(s), resource allocation for the relays (subcarriers and power), link asymmetry and imperfection in CSI. All the previous studies lacked in including one of those properties. Here we jointly consider all of them. In particular, since the transmission durations of the source-to-relay link (first hop) and relay-to-destination link (second hop) are not necessary equal, we investigate the RRA problem in this setting and advocate a scheme to solve the joint relay set selection for cooperation in addition to asymmetry, subcarrier and power allocations. The target of our proposed scheme is to enhance the total system throughput when only estimated CSI is available at the source. In this work, relays are deployed for extending cell coverage, so we do not consider a direct link from source to destination. One particular case of our proposed scheme is the symmetric resource allocation, when the transmission durations for two hops are the same. Since the channel capacity in the presence of imperfect CSI is unknown, we use the conditional capacity expectation as the performance metric [17]. We propose a relay selection method and a subcarrier allocation scheme, where a set of relays that can obtain the best data rate for the link is selected. Power is allocated to the source and relays under per-node constraints, which is more realistic than the scheme, e.g., in [10] where only the sum of the whole system power was considered. To the best of our knowledge, such joint optimization for asymmetric two hop OFDMA networks with imperfect CSI consideration has not been reported so far. The key contributions of the paper can be divided into four folders:

1) Problem formulation: we formulate the problem as a joint optimization problem for asymmetric two hop OFDMA networks including a relay set selection, subcarrier, power and asymmetry allocations in the presence of imperfect knowledge of the CSI;
2) Resource allocation algorithms: We solve the optimization problem by using analytical tools. Numerical examples are given to illustrate theoretical expressions;
3) Compared with other previous work, we consider all the fundamental properties jointly: the selection of the relay(s), resource allocation for the relays (subcarriers and power), link asymmetry and imperfection in CSI, which have never been reported.
4) Performance Evaluation: The system performance of proposed scheme is compared with some other recent proposed scheme in this area. We show that the proposed scheme has superior performance over other existed work.

The rest of this paper is organized as follows. Section II describes the relay-assisted OFDMA cooperative wireless networks and formulates the problem. We only consider the downlink in this work, but it can be extended further to the uplink case. In Section III, the proposed resource allocation scheme is presented. We demonstrate the benefits of our proposed algorithm in section IV and finally conclude the article in Section V.

II. System Model and Problem Formulation

A. System Model and Assumptions

This paper investigates the problem of RRA in the cooperative relay OFDMA network. We consider our system as a two-hop time-division duplex downlink relay system. The whole system consists of a source, e.g., access point (AP), a destination node, e.g., mobile terminal (MT) and several relays. In the first hop, AP delivers information data to a cluster of decode-and-forward (DF) relays. In the second hop, relays cooperate to transmit the information data to the MT, so the spatial diversity gain can be achieved (the relays are assumed to be far enough from each other). The estimated CSI is assumed to be known at the receiver by using the estimator and then fed back to the transmitter perfectly. We also assume that channel estimation error pertains to the amplitude of the correct channel gain, while the phase of the channel gain can be perfectly obtained. As a result, information about the channel gain with an estimation error is available to both the transmitter and the receiver. The AP acts as a central controller to carry out all resource allocation related operations based on the imperfect CSI from the MT.

In this work, we assume that there are total Z relays in the networks, and the selected relay cluster $\mathcal{K} = \{1, ..., k, ..., K\}$ contains K potential half-duplex relays. The presented relay-assisted collaborative OFDMA network is as shown in Fig. 1 when $K = 2$.

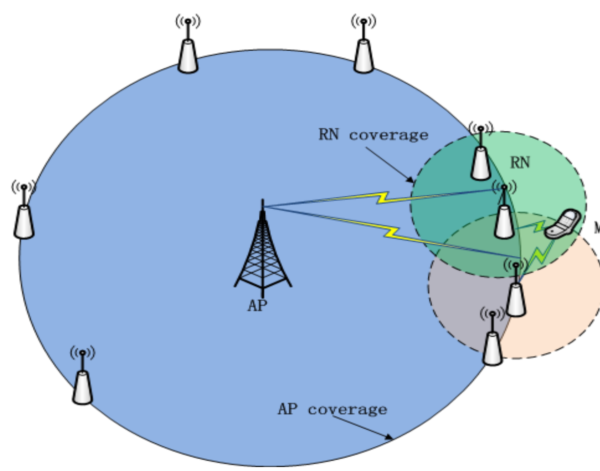

Figure 1. Wireless cooperative relay networks

B. Problem Formulation

Let x be the transmitted data from transmitter to receiver and P the transmit power. Excluding the effect of the path loss, the received data at receiver can be modeled as

$$y = h\sqrt{P}x + n, \quad (1)$$

and we have

$$h = \hat{h} + \tilde{h}, \quad (2)$$

where $\hat{h}$ is the estimated channel function and $\tilde{h}$ is the independent estimation error which can be modeled as a zero mean Gaussian random variable with variance $\sigma^2_{\tilde{h}}$. As such, the imperfect CSI h follows $\mathcal{CN}(\hat{h}, \sigma^2_{\tilde{h}})$. n is the additive noise which can be also modeled as complex Gaussian random variable with variance σ^2_n. Therefore, the square of imperfect CSI h follows a noncentral chi-square probability density function (PDF) given by [18]

$$f(G|\hat{G}) = \frac{1}{\sigma^2_{\tilde{h}}} e^{-\frac{\hat{G}+G}{\sigma^2_{\tilde{h}}}} \mathcal{J}_0\Big(2\sqrt{\frac{\hat{G}G}{\sigma^4_{\tilde{h}}}}\Big) \quad (3)$$

where we denote $G = |h|^2$,$\hat{G} = |\hat{h}|^2$. $\mathcal{J}_0$ is the 0th order modified Bessel Function of the first kind.

In our proposed system model, we suppose h^i is the channel transfer function from transmitter to receiver and we assume the channel is static within a time slot. For example, $\hat{h}^i_{s,k}$ means the channel estimate from AP s to relay node (RN) k over OFDM subcarrier i and $\hat{h}^j_{k,d}$ means the channel estimate from RN k to destination d over OFDM subcarrier j. We have the channel gain of the first hop $\hat{G}^i_{s,k} = |\hat{h}^i_{s,k}|^2$ and the second hop $\hat{G}^j_{k,d} = |\hat{h}^j_{k,d}|^2$. The noise variance for two hops are σ^2_k and σ^2_d. The variance of related estimation error for two hops are $\sigma^2_{\tilde{h},k}$ and $\sigma^2_{\tilde{h},d}$ and we assume $\sigma^2_{\tilde{h}} = \sigma^2_{\tilde{h},k} = \sigma^2_{\tilde{h},d}$. We denote the transmit power assigned to the subcarrier i for transmitting data as P^i. In this work, we do not consider the direct link from AP to MT due to distance or obstacles. This assumption is reasonable in the case that the RNs are deployed for cell extension. One RN k occupies subcarrier i in the first hop and j in the second hop. In this work, we assume that the transmission durations for the first hop and second hop are allowed to differ. We denote these durations as T_1 and T_2. Therefore, in the first hop, the data rate of the first hop is determined by the minimum rate of each link between AP and and selected RNs. Since the transmitter only knows the CSI conditioned on the feedback of the receiver, we could obtain the expected achievable throughput of the first hop as follows

$$R^{\mathcal{I}}_{s,\mathcal{K}} = \min_{k\in\mathcal{K}} \Big\{ \mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}} \big[\frac{T_1}{T} log(1 + \sum_{i=1}^{M} \omega^i_{s,k}\rho_k P^i_{s,k}\gamma^i_{s,k})\big] \Big\}, \quad (4)$$

where $\gamma^i_{s,k} = \frac{L_{s,k}G^i_{s,k}}{\sigma^2_k}$, $\hat{\gamma}^i_{s,k} = \frac{L_{s,k}\hat{G}^i_{s,k}}{\sigma^2_k}$ and $T = T_1 + T_2$. Here L stands for the path loss factor of a link in question. The notation $\mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}}$ means expectation with respect to $\gamma^i_{s,k}$ conditioned on $\hat{\gamma}^i_{s,k}$. There are in total M subcarriers and $\mathcal{I}$ is the subcarrier set which contains the subcarriers that are allocated to the selected RNs at first hop. We further refer the link throughput and its expectation interchangeably for simplicity. ρ_k indicates that whether RN k is chosen for subcarrier allocation, so we obtain

$$\rho_k = \begin{cases} 1 & \text{if } k \text{ is chosen for relaying,} \\ 0 & \text{otherwise.} \end{cases}$$

We also define ω is the indicator whether certain subcarrier is assigned to RN k, which is,

$$\omega^i_{s,k} = \begin{cases} 1 & \text{if } i \text{ is assigned to } k \text{ at first hop,} \\ 0 & \text{otherwise.} \end{cases}$$

For the second hop, it is assumed that the RNs are perfectly synchronized and transmitted at the same time. Therefore, the link throughput can be calculated as [19]

$$R^{\mathcal{J}}_{\mathcal{K},d} = \mathbb{E}_{\gamma^j_{k,d}|\hat{\gamma}^j_{k,d}} \Big[\frac{T_2}{T} log\big(1 + \sum_{j=1}^{M}\sum_{k=1}^{K} \omega^j_{k,d}\rho_k P^j_{k,d}\gamma^j_{k,d}\big)\Big], \quad (5)$$

where $\gamma^j_{k,d} = \frac{L_{k,d}G^j_{k,d}}{\sigma^2_d}$ and $\hat{\gamma}^j_{k,d} = \frac{L_{k,d}\hat{G}^j_{k,d}}{\sigma^2_d}$. $\mathcal{J}$ is the subcarrier set which contains the subcarriers that are allocated to the selected RNs at second hop. For indicator $\omega^j_{k,d}$, we also have

$$\omega^j_{k,d} = \begin{cases} 1 & \text{if } j \text{ is assigned to } k \text{ at second hop,} \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, the total achieved end-to-end throughput of source s to destination d through RN set $\mathcal{K}$ is [20]

$$R_{sd} = min\ \Big\{R^{\mathcal{I}}_{s,\mathcal{K}}, R^{\mathcal{J}}_{\mathcal{K},d}\Big\}. \quad (6)$$

To proceed, we can formulate our problem as

$$max\ \ R_{sd}, \quad (7)$$

subject to

$$\begin{aligned} & T = T_1 + T_2 \\ & \sum_{i=1}^{M}\sum_{k=1}^{K} \omega^i_{s,k}P^i_{s,k} \le P_{s,max} \\ & \sum_{j=1}^{M} \omega^j_{k,d}P^j_{k,d} \le P_{k,max} \\ & \sum_{k=1}^{K} \omega^i_{s,k} = 1, \omega^i_{s,k} \in \{0,1\} \\ & \sum_{k=1}^{K} \omega^j_{k,d} = 1, \omega^j_{k,d} \in \{0,1\} \end{aligned} \quad (8)$$

where $P_{s,max}$ is the maximum transmit power of AP and $P_{k,max}$ is the maximum power of RN k. Therefore, our goal is to optimize the relay set and subcarrier, power and asymmetry allocations which satisfy the problem (7).

It can be deduced that (6) can achieve minimum only when $R^{\mathcal{I}}_{s,\mathcal{K}} = R^{\mathcal{J}}_{\mathcal{K},d}$ [21]. Thus, (7) can be rearranged to

$$\arg\max\ \Big(R^{\mathcal{I}}_{s,\mathcal{K}} + R^{\mathcal{J}}_{\mathcal{K},d}\Big), \quad (9)$$

subject to conditions in (8) and

$$R^{\mathcal{I}}_{s,\mathcal{K}} = R^{\mathcal{J}}_{\mathcal{K},d}. \quad (10)$$

$$\mathcal{L}(\mathbf{P},\boldsymbol{\omega},\boldsymbol{\rho},\boldsymbol{\lambda},\boldsymbol{\mu}) = \Big(R^{\mathcal{I}}_{s,\mathcal{K}}+R^{\mathcal{J}}_{\mathcal{K},d}\Big)-\lambda_s(\sum_{i=1}^{M}\sum_{k=1}^{K}\omega^i_{s,k}P^i_{s,k}-P_{s,max})-\sum_{k=1}^{K}\lambda_{k,d}(\sum_{j=1}^{M}\omega^j_{k,d}P^j_{k,d}-P_{k,max})-\mu(R^{\mathcal{I}}_{s,\mathcal{K}}-R^{\mathcal{J}}_{\mathcal{K},d}), \quad (11)$$

III. Resource Allocation Scheme

In this section, we introduce adaptive algorithms to solve the problem (9). Although the resource allocation problem is combinatorial in nature with a nonconvex structure, it has been shown in [24] that the duality gap of the optimization problem is assumed to be negligible under the condition of time-sharing regardless of its convexity. As such, it can be solved in the dual domain and the solution is asymptotically optimal. The Lagrangian of problem (9) is as shown in (11) [22], where $\mathbf{P} = \{P^i_{s,k}, P^j_{k,d}\}$ is the set of power allocation, $\boldsymbol{\omega} = \{\omega^i_{s,k}, \omega^j_{k,d}\}$ denotes the subcarrier allocation, and $\boldsymbol{\rho} = \{\rho_k\}$ is the relay assignment. The variables $\boldsymbol{\lambda} = \{\lambda_s, \lambda_{k,d}\}$ and $\boldsymbol{\mu} = \{\mu\}$ are Lagrange multipliers. It can be derived that $\lambda_s, \lambda_{k,d} \geq 0$ and $\mu \in (-1, 1)$. The Lagrange dual function can be written as

$$g(\boldsymbol{\lambda},\boldsymbol{\mu}) = \max \mathcal{L}(\mathbf{P},\boldsymbol{\omega},\boldsymbol{\rho},\boldsymbol{\lambda},\boldsymbol{\mu}). \quad (12)$$

Since we assume the number of subcarriers are sufficiently large, the duality gap between the primal problem and dual function is negligible [24]. It can be noticed that in [24] and [11], 32 subcarriers are enough to make duality gap negligible. Considering in the specifications of LTE, the number of subcarrier is much larger than 32, thus, our assumption is realistic. Consequently, we can tackle the problem (7) by minimizing the dual function

$$\min \ g(\boldsymbol{\lambda},\boldsymbol{\mu}). \quad (13)$$

A. Evaluating Dual Variable

As a dual function is always convex [22], many methods can be used to minimize $g(\boldsymbol{\lambda},\boldsymbol{\mu})$ and find the dual point with guaranteed convergence. We follow the subgradient method in [24] to derive the subgradient $g(\boldsymbol{\lambda},\boldsymbol{\mu})$ with the optimal power allocation $\mathbf{P}^*$ which will be presented in the following subsection.

Algorithm 1 Evaluating Dual Variable

Initialize $\boldsymbol{\lambda}^0$ and $\boldsymbol{\mu}^0$

while (!Convergence) **do**

  Obtain $g(\boldsymbol{\lambda}^a,\boldsymbol{\mu}^a)$ at the ath iteration;

  Update a subgradient for $\boldsymbol{\lambda}^{a+1}$ and $\boldsymbol{\mu}^{a+1}$, by $\boldsymbol{\lambda}^{a+1} = \boldsymbol{\lambda}^a + \upsilon^a \triangle\boldsymbol{\lambda}$ and $\boldsymbol{\mu}^{a+1} = \boldsymbol{\mu}^a + \upsilon^a \triangle\mu$;

end while

where $\triangle\boldsymbol{\lambda} = \{\triangle\lambda_s, \triangle\lambda_{1,d}, ...\triangle\lambda_{K,d}\}$, $\triangle\lambda_s$, $\triangle\lambda_{k,d}$ and $\triangle\mu$ can be expressed as

$$\begin{aligned} \triangle\lambda_s &= P_{s,max} - \sum_{i=1}^{M}\sum_{k=1}^{K}(P^i_{s,k})^* \\ \triangle\lambda_{k,d} &= P_{k,max} - \sum_{j=1}^{M}(P^j_{k,d})^* \\ \triangle\mu &= (R^{\mathcal{I}}_{s,\mathcal{K}})^* - (R^{\mathcal{J}}_{\mathcal{K},d})^*. \end{aligned} \quad (14)$$

Here, υ^a is the stepsize which can be chosen following a diminishing step size rule according to [23] and a is the number of iterations. The subgradient algorithm in Algorithm 1 is guaranteed to converge to the global optimal $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$. The computational complexity of Algorithm 1 is polynomial in the number of dual variable $K+1$ [24]. (12) can be viewed as nonlinear integer programming problem, whose optimal solution requires high computational cost. Therefore, we are aiming to solve the optimization problem by solving the three sub-problems, which are relay set selection, subcarrier, power and asymmetry allocations. Firstly we introduce a power allocation scheme with provision for link asymmetry.

B. Power Allocation Scheme

By assuming the relay selection and subcarrier allocation have been done (i.e., relays and subcarriers can be assigned randomly at the beginning), the obtained time slot for each hop can be achieved by using Karush-Kuhn-Tucker (KKT) conditions [22]. This results in

$$T_1 = \frac{1-\mu^*}{2}T \quad (15)$$

$$T_2 = \frac{1+\mu^*}{2}T \quad (16)$$

The proof is given in Appendix A. In order to obtain the optimal solution of power allocation, we are aiming to solve the problem (11) over variables $P^i_{s,k}$ and $P^j_{k,d}$. However, from (4) and (5), we see that problem (11) involves the conditional expectation of achievable throughput with respect to the estimated CSI. Applying KKT conditions [22], we could obtain the optimal power allocation schemes by solving the following equation numerically by using, e.g., matlab:

$$\begin{aligned} &\frac{\alpha^i_{s,k}}{P^i_{s,k}}\Big(\frac{\sigma_k^2\beta^i_{s,k}}{L_{s,k}P^i_{s,k}}\Big)^{\alpha^i_{s,k}} e^{\frac{\sigma_k^2\beta^i_{s,k}}{L_{s,k}P^i_{s,k}}}\Gamma\Big(-\alpha^i_{s,k}, \frac{\sigma_k^2\beta^i_{s,k}}{L_{s,k}P^i_{s,k}}\Big) \\ &= \frac{2\lambda_s}{(1-\mu)^2}. \end{aligned} \quad (17)$$

Here $\Gamma(a,b)$ is the incomplete Gamma function, $\alpha^i_{s,k} = (\eta^i_{s,k}+1)^2/(2\eta^i_{s,k}+1)$ is the Gamma shape parameter with $\eta^i_{s,k} = \hat{G}^i_{s,k}/\sigma^2_{\tilde{h}}$ and $\beta^i_{s,k} = \alpha^i_{s,k}/(\hat{G}^i_{s,k}+\sigma^2_{\tilde{h}})$ is Gamma PDF

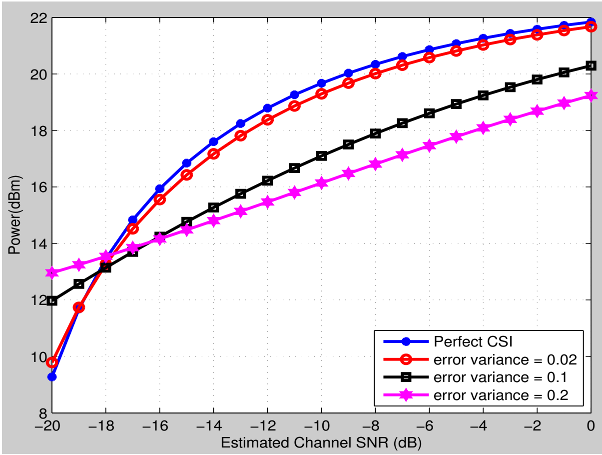

Figure 2. One example of power allocation as function of estimated channel SNR for various value of $\sigma^2_{\tilde{h}}$.

rate parameter. Similarly, for the cooperation phase, the RN power allocation is obtained by solving

$$\frac{\alpha^j_{k,d}}{P^j_{k,d}}\left(c_1\beta^j_{k,d}\right)^{\alpha^j_{k,d}}e^{c_1\beta^j_{k,d}}\Gamma\left(-\alpha^j_{k,d},c_1\beta^j_{k,d}\right)=\frac{2\lambda_{k,d}}{(1+\mu)^2}, \quad (18)$$

where $\alpha^j_{k,d}=(\eta^j_{k,d}+1)^2/(2\eta^j_{k,d}+1)$ with $\eta^j_{k,d}=\hat{G}^j_{k,d}/\sigma^2_{\tilde{h}}$ and $\beta^j_{k,d}=\alpha^j_{k,d}/(\hat{G}^j_{k,d}+\sigma^2_{\tilde{h}})$. Here we have $c_1=\frac{\sigma^2_d+\sum^K_{m=1,m\neq k}P_{m,d}L_{m,d}G_{m,d}}{L_{k,d}P^j_{k,d}}$. $P_{m,d}$ and $G_{m,d}$ is the power allocation and channel gain from relay m to MT d. Proof is given in Appendix B. By using an approximation method like e.g. in [28], we are able to obtain the power allocation with imperfect CSI. One could notice during the resource allocation process, when relay selection and subcarrier allocation have been done, the essential parameters are then known. Thus, we can use some mathematical tool to solve (17) and (18) numerically.

When symmetric links are considered, by following the same procedure, we could obtain the optimal power allocation schemes by solving the following equations numerically:

$$\frac{\alpha^i_{s,k}}{P^i_{s,k}}\Big(\frac{\sigma^2_k\beta^i_{s,k}}{L_{s,k}P^i_{s,k}}\Big)^{\alpha^i_{s,k}}e^{\frac{\sigma^2_k\beta^i_{s,k}}{L_{s,k}P^i_{s,k}}}\Gamma\Big(-\alpha^i_{s,k},\frac{\sigma^2_k\beta^i_{s,k}}{L_{s,k}P^i_{s,k}}\Big)=\frac{\lambda_s}{1-\mu}. \quad (19)$$

$$\frac{\alpha^j_{k,d}}{P^j_{k,d}}\left(c_1\beta^j_{k,d}\right)^{\alpha^j_{k,d}}e^{c_1\beta^j_{k,d}}\Gamma\left(-\alpha^j_{k,d},c_1\beta^j_{k,d}\right)=\frac{\lambda_{k,d}}{1+\mu}, \quad (20)$$

The proof can be found in Appendix C. One example is shown in Fig. 2 where different values of $\sigma^2_{\tilde{h}}$ are considered. We can see that when the estimation error is relatively small, the power allocation in the presence of imperfect CSI is very close to the one when perfect CSI is assumed at the AP.

C. Opportunistic Relay Selection (ORS)

The considered relay selection problem in this work, unlike some traditional single relay selection algorithms in [11] and [25], is a multiple RN selection problem. The proposed algorithm is to select K RNs to form a cluster that can maximize the achieved throughput in (6) based on the imperfect CSI.

When assuming the subcarrier and power allocations have been done, we can rewrite (11) as expressed in (17). Also, by applying the KKT condition, the RN is selected according to the rule expressed in (18).

In case of symmetric links, we have the optimal relay set as

$$\mathcal{K}^*=\arg\max_k\ \Bigg(\min_{k\in\mathcal{K}}\Big\{(1-\mu^*)\mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}}\Big[\frac{P_{s,k}\gamma_{s,k}}{1+P_{s,k}\gamma_{s,k}}\Big]\Big\}$$
$$+(1+\mu^*)\mathbb{E}_{\gamma^j_{k,d}|\hat{\gamma}^j_{k,d}}\Big[\frac{P_{k,d}\gamma_{k,d}}{1+\sum^K_k P_{k,d}\gamma_{k,d}}\Big]\Bigg), \quad (19)$$

Since we know that $\hat{\gamma}^i_{s,k}=\frac{L_{s,k}\hat{G}^i_{s,k}}{\sigma^2_k}$, the channel SNR $\gamma_{s,k}$ conditioned on $\hat{\gamma}^i_{s,k}$ is also a non-central Chi-squared distributed random variable with PDF

$$f(\gamma^i_{s,k}|\hat{\gamma}^i_{s,k})=\frac{1}{\nu^i_{s,k}}e^{-\frac{\hat{\gamma}^i_{s,k}+\gamma^i_{s,k}}{\nu^i_{s,k}}}\mathcal{J}_0\left(2\sqrt{\frac{\hat{\gamma}^i_{s,k}\gamma^i_{s,k}}{(\nu^i_{s,k})^2}}\right) \quad (20)$$

$$f(\gamma^i_{s,k}|\hat{\gamma}^j_{k,d})=\frac{1}{\nu^j_{k,d}}e^{-\frac{\hat{\gamma}^j_{k,d}+\gamma^j_{k,d}}{\nu^j_{k,d}}}\mathcal{J}_0\left(2\sqrt{\frac{\hat{\gamma}^j_{k,d}\gamma^j_{k,d}}{(\nu^j_{k,d})^2}}\right) \quad (21)$$

where $\nu^i_{s,k}=\sigma^2_k/\sigma^2_{\tilde{h}}$ and $\nu^j_{k,d}=\sigma^2_d/\sigma^2_{\tilde{h}}$. Consequently, following the same procedure as in Appendix B, one can arrive at

$$\mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}}\Big[\frac{P^i_{s,k}\gamma^i_{s,k}}{1+P^i_{s,k}\gamma^i_{s,k}}\Big]$$
$$=\psi^i_{s,k}\Big(\frac{\theta^i_{s,k}}{P^i_{s,k}}\Big)^{\psi^i_{s,k}}e^{\frac{\theta^i_{s,k}}{P^i_{s,k}}}\Gamma\Big(-\psi^i_{s,k},\frac{\sigma^2_k\theta^i_{s,k}}{L_{s,k}P^i_{s,k}}\Big), \quad (22)$$

$$\mathbb{E}_{\gamma^j_{k,d}|\hat{\gamma}^j_{k,d}}\Big[\frac{P^j_{k,d}\gamma^i_{k,d}}{1+\sum^K_k P^j_{k,d}\gamma^j_{k,d}}\Big]$$
$$=\psi^j_{k,d}\big(c_2\theta^j_{k,d}\big)^{\psi^j_{k,d}}e^{c_2\theta^j_{k,d}}\Gamma\big(-\psi^j_{k,d},c_2\theta^j_{k,d}\big), \quad (23)$$

where $\psi^i_{s,k}=(\zeta^i_{s,k}+1)^2/(2\zeta^i_{s,k}+1)$ with $\zeta^i_{s,k}=\hat{\gamma}^i_{s,k}/\nu^i_{s,k}$ and $\theta^i_{s,k}=\zeta^i_{s,k}/(\hat{\gamma}^i_{s,k}+\nu^i_{s,k})$. Similarly, $\psi^j_{k,d}=(\zeta^j_{k,d}+1)^2/(2\zeta^j_{k,d}+1)$ with $\zeta^j_{k,d}=\hat{\gamma}^j_{k,d}/\nu^j_{k,d}$ and $\theta^j_{k,d}=\zeta^j_{k,d}/(\hat{\gamma}^j_{k,d}+\nu^j_{k,d})$. Also we have $c_2=(1+\sum^K_{m=1,m\neq k}P_{m,d}\gamma_{m,d})/P^j_{k,d}$, and $P_{m,d}$ and $\gamma_{m,d}$ are the power allocation and channel SNR from relay m to MT. The optimal value of $\mathbf{P}$ can be expressed as in (17) and (18). Thus, (18) can be viewed as a multi-objective optimization problem, which aims at obtaining a trade-off of the throughput of the first hop and second hop. (18) also acts as the termination criteria for the whole RRA scheme. The ORS scheme is depicted in Algorithm 2. Therefore, the relay selection strategy is

$$\rho_k=\begin{cases}1 & \text{if } k\in\mathcal{K}^*,\\ 0 & \text{otherwise.}\end{cases}$$

$$\begin{aligned}\mathcal{L}(\mathbf{P},T,\boldsymbol{\omega},\boldsymbol{\rho},\boldsymbol{\lambda}) =& \min_{k\in\mathcal{K}}\Big\{\mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}}[\frac{T_1}{T}log(1+\sum_{i=1}^{M}\omega^i_{s,k}\rho_k P^i_{s,k}\gamma^i_{s,k})]\Big\} + \mathbb{E}_{\gamma^j_{k,d}|\hat{\gamma}^j_{k,d}}[\frac{T_2}{T}log(1+\sum_{i=1}^{M}\sum_{k=1}^{K}\omega^i_{s,k}\rho_k P^j_{k,d}\gamma^j_{k,d})]\\ &-\mu\Big(\min_{k\in\mathcal{K}}\Big\{\mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}}[\frac{T_1}{T}log(1+\sum_{i=1}^{M}\omega^i_{s,k}\rho_k P^i_{s,k}\gamma^i_{s,k})]\Big\} - \mathbb{E}_{\gamma^j_{k,d}|\hat{\gamma}^j_{k,d}}[\frac{T_2}{T}log(1+\sum_{i=1}^{M}\sum_{k=1}^{K}\omega^i_{s,k}\rho_k P^j_{k,d}\gamma^j_{k,d})]\Big)\\ &+\lambda_s(\sum_{i=1}^{M}\sum_{k=1}^{K}\omega^i_{s,k}P^i_{s,k} - P_{s,max}) - \sum_{k=1}^{K}\lambda_{k,d}(\sum_{j=1}^{M}\omega^j_{k,d}P^j_{k,d} - P_{k,max}).\end{aligned} \tag{17}$$

$$\mathcal{K}^* = \arg\max_{k} \left(\min_{k\in\mathcal{K}}\Big\{\frac{(1-\mu^*)^2}{2}\mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}}\Big[\frac{P_{s,k}\gamma_{s,k}}{1+P_{s,k}\gamma_{s,k}}\Big]\Big\} + \frac{(1+\mu^*)^2}{2}\mathbb{E}_{\gamma^j_{k,d}|\hat{\gamma}^j_{k,d}}\Big[\frac{P_{k,d}\gamma_{k,d}}{1+\sum_k^K P_{k,d}\gamma_{k,d}}\Big]\right), \tag{18}$$

Algorithm 2 ORS

Definition
$\mathcal{Z}$ is the set of all Z RNs.
$\mathcal{K}$ is the set of selected K RNs;
$C_{\mathcal{K}} = 0$ for $\forall k \in \mathcal{K}$;
sort the set of RN in the descending order according to its overall path loss;
while !satisfy (18) **do**
  for z = 1 to Z **do**
    add RN z to $\mathcal{K}$ according to its order;
    do subcarrier and power allocation;
    calculates the value of $C_{\mathcal{K}}$ according to the right-hand side of (18)
    find z satisfying (18),$\forall\ z \in \mathcal{Z}$;
  end for
end while

D. Optimal Subcarrier Allocation (OSA)

The objective of the subcarrier allocation strategy is to assign subcarriers to a given RN that can obtain best throughput performance. Following the same procedure as the relay selection, we can obtain the subcarrier allocation criteria as follows:

$$\mathcal{I}^* = \arg\max \left(\min_{k\in\mathcal{K}}\Big\{\frac{(1-\mu^*)^2}{2}\mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}}\big[\frac{P^i_{s,k}\gamma^i_{s,k}}{1+P^i_{s,k}\gamma^i_{s,k}}]\Big\}\right) \tag{24}$$

$$\mathcal{J}^* = \arg\max\Big\{\frac{(1+\mu^*)^2}{2}\mathbb{E}_{\gamma^j_{k,d}|\hat{\gamma}^j_{k,d}}\big[\frac{P^j_{k,d}\gamma^j_{k,d}}{1+\sum_k^K P^j_{k,d}\gamma^j_{k,d}}]\Big) . \tag{25}$$

Similarly, in case of a symmetric relay network, the subcarrier allocation can be obtained by

$$\mathcal{I}^* = \arg\max \left(\min_{k\in\mathcal{K}}\Big\{(1-\mu^*)\mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}}\big[\frac{P^i_{s,k}\gamma^i_{s,k}}{1+P^i_{s,k}\gamma^i_{s,k}}]\Big\}\right) \tag{26}$$

$$\mathcal{J}^* = \arg\max\Big\{(1+\mu^*)\mathbb{E}_{\gamma^j_{k,d}|\hat{\gamma}^j_{k,d}}\big[\frac{P^j_{k,d}\gamma^j_{k,d}}{1+\sum_k^K P^j_{k,d}\gamma^j_{k,d}}]\Big\}, \tag{27}$$

where channel SNR $\gamma^i_{s,k} = \frac{L_{s,k}G^i_{s,k}}{\sigma^2_k}$ and $\gamma^j_{k,d} = \frac{L_{k,d}G^j_{k,d}}{\sigma^2_d}$. The detailed procedure of OSA is shown in Alg. 3. Therefore, the OSA indicator for the first hop and second hop can be expressed as

$$\omega_i = \begin{cases} 1 & \text{if } i \in \mathcal{I}^*, \\ 0 & \text{otherwise.} \end{cases}$$

$$\omega_j = \begin{cases} 1 & \text{if } j \in \mathcal{J}^* \ , \\ 0 & \text{otherwise.} \end{cases}$$

Algorithm 3 OSA

Definition
c_1: the set of M subcarriers at first hop;
c_2: the set of M subcarriers at second hop;
while !satisfy (24) and (25) **do**
  sort c_1 and c_2 in the descending order according to the fast fading gain.
  for m = 1 to M **do**
    find subcarrier set $\mathcal{I}$ for the first hop that satisfy (24);
    find subcarrier set $\mathcal{J}$ for the first hop that satisfy (25);
  end for
end while

E. Joint Relay, Subcarrier and Power Allocation

We have described the algorithms for relay selection, subcarrier and power allocation in the previous section. The four subproblems presented are interconnected hierarchically. Combining the above three phases together with asymmetric time design, we can obtain suboptimal solution for (7) when number of subcarriers is sufficiently large. The flow chart of the whole algorithm is shown in Fig. 3. We can see these four steps are conducted in alternating fashion until the convergence is reached.

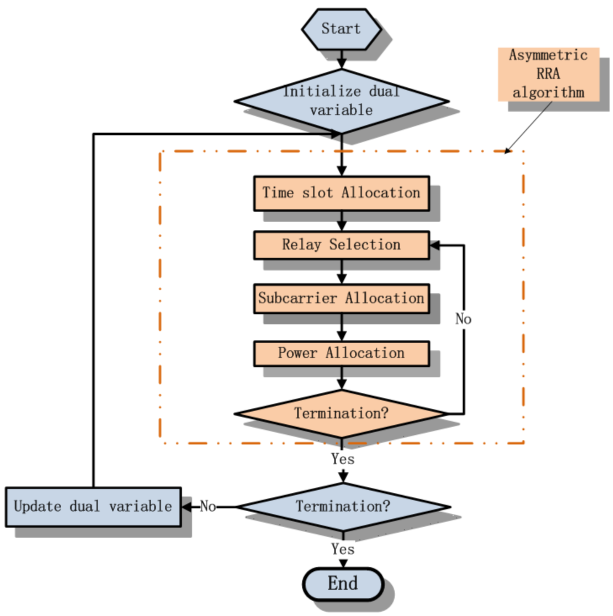

Figure 3. Algorithm flow chart

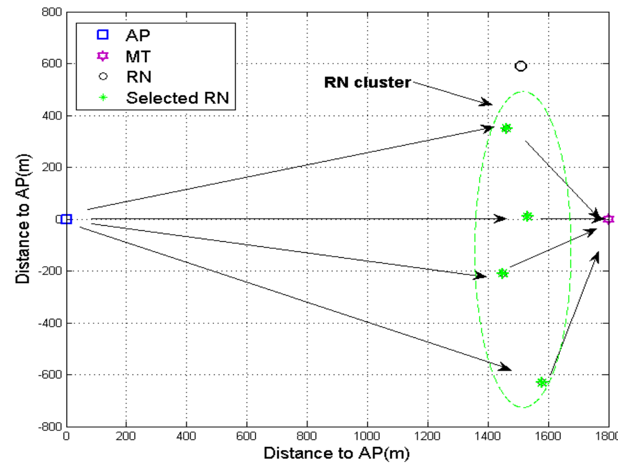

Figure 4. Relay node distribution and 4 RNs are selected

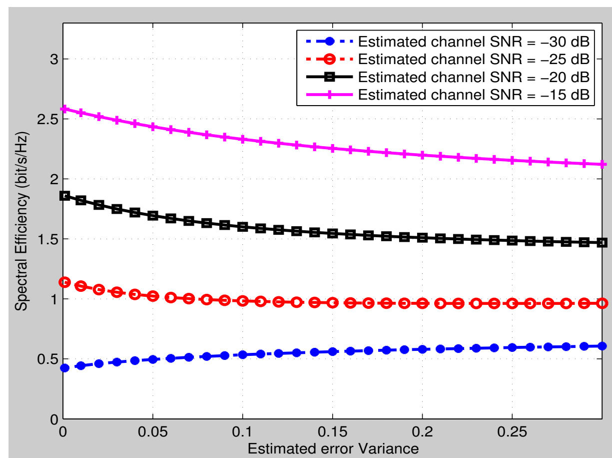

Figure 5. One example of impact of $\sigma_{\tilde{h}}^2$ on the system bandwidth Efficiency

IV. Performance Evaluation

Simulations are presented to illustrate the performance of the proposed algorithms. It is assumed that five RNs are located between AP and MT, and MT is 1.8km away from AP. One example of RN distribution is shown in Fig. 4, when four RNs are selected for transmission. The Stanford University SUI-3 channel model is used and modified to include multipath effects [29]. The central frequency is 1.9GHz. We use the 3-tap channel and signal fading follows Rician distribution. We choose the number of subcarriers N to be 32, so the duality gap can be ignored [11]. Flat quasi-static fading channels are adopted, hence the channel coefficients are assumed to be constant during a complete frame, and can vary from a frame to another independently. The noise variance of the two hops are set to be 1 for simplicity. The path loss factor varies according to the different distances from RNs to AP and MT. If the distance between RN and AP or RN and MT is shorter than the break point $d_{BP} = 100m$, the path loss exponent is fixed to 2, otherwise it is 3.5. The maximum transmit power of AP and RN are set to 40 dBm and 20 dBm, respectively. If not otherwise stated, the channel estimator with an error variance $\sigma_{\tilde{h}}^2 = 0.02$ is assumed at the receiver. There are 1000 simulation trials and we consider the average system performance of these trials.

We demonstrate our results, labeled as 'Proposed ARRA' and 'Proposed RRA', comparing them with the performance of recently reported schemes:

1) The asymmetric resource allocation scheme in [12](ARA);
2) The water-filling power allocation and proposed subcarrier allocation scheme with relay selection (Waterfilling);
3) The modified proportional allocation scheme in [10] with fairness consideration (Fairness SA with Asymmetric/Symmetric link);

At first, the impact of CSI error variance $\sigma_{\tilde{h}}^2$ to the system spectral efficiency is depicted in Fig. 5. We can notice that the accuracy of the estimator can lead to up to 20% differences on the spectral efficiency when the estimated channel SNR is 20 dB. If we use an estimator with variance $\sigma_{\tilde{h}}^2 = 0.02$, it can result in around 5% difference on the systems performance.

Fig. 6 demonstrates the impact of maximum transmit power of AP on the system spectral efficiency. We denote $D_{s,d}$ as the distance between AP and MT, and $D_{s,k}$ as the distance between AP and RNs. In Fig. 6, we have $D_{s,d} = 1800\ m$ and $D_{s,k}$ from $1500\ m$ to $1600\ m$. The considered channel SNR at the RN k is varied from $\gamma_{s,k} = -20dB$ to $\gamma_{s,k} = -30dB$ and at MT d it is varied from $\gamma_{k,d} = -15dB$ to $\gamma_{k,d} = -25dB$. The 'ES' stands for the exhaustive search algorithm which provides the optimal solution. It can be seen that the proposed ARRA scheme achieves the best performance and its performance is very close to the optimal one. The performance gain in terms of spectral efficiency is pretty evident, reaching 100% when comparing to Fairness SA scheme. Generally we can observe that if different time duration for different hop and multi-relay selection is considered, the throughput performance is better than the one with equal time duration for different hop. It can also be noticed that if ARA is used as the resource allocation scheme (instead of our proposed scheme),

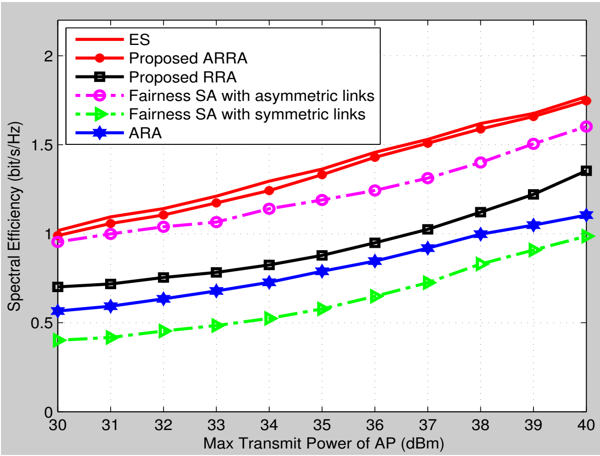

Figure 6. Impact of maximum transmit power $P_{s,max}$ on system bandwidth Efficiency

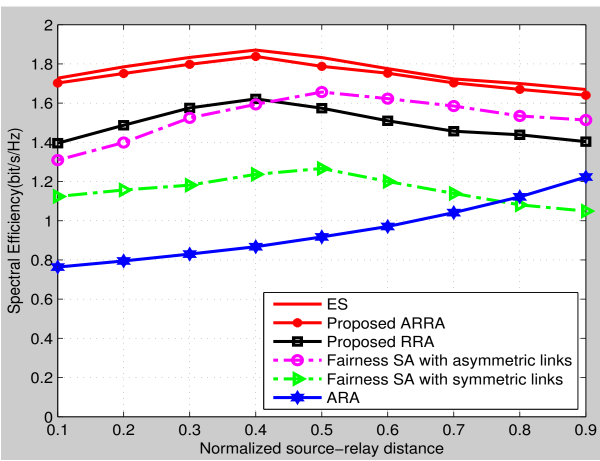

Figure 7. Impact of distance between AP and RN on the system bandwidth efficiency, maximum AP power is 35 dB, maximum RN power is 20 dB

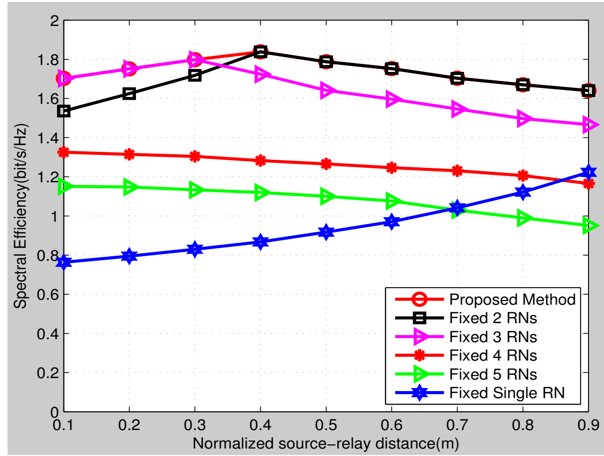

Figure 8. Impact of distance between AP and RN on system spectral efficiency, comparison with fix-number of relay without relay selection.

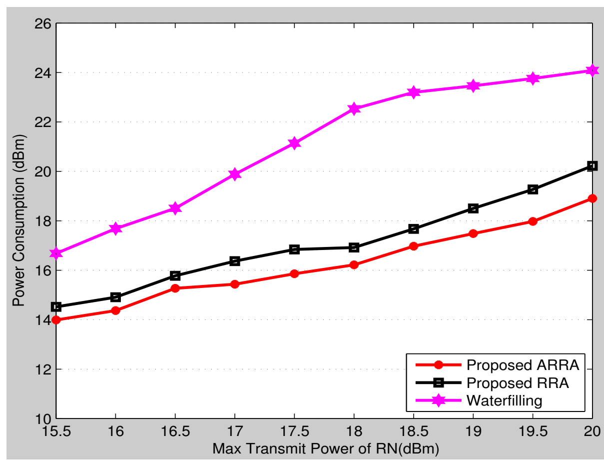

Figure 9. Power consumption of all RNs

the throughput performance is comparable to Fairness SA with a symmetric link. Another performance gain can be seen in power consumption. We can observe that with a fixed data rate requirement, our proposed scheme provides a significant power saving gain over others. For instance, at the level of 1 bit/s/Hz spectral efficiency our proposed scheme can reach a power saving of around 10 dB compared to the other schemes, i.e., ARA scheme.

Fig. 7 depicts the impact of the distance between AP and RN on the system throughput. The optimal result obtained by exhaustive searching is also presented for comparison. The distance between AP and RN is normalized to the distance between AP and MT, and varies from 0.1 to 0.9. In Fig. 7, we fix the maximum AP power to $P_{s,max} = 40\ dBm$ and maximum RN power to $P_{k,max} = 20\ dBm$. From Fig. 7 it can be seen that the proposed algorithm with asymmetric allocations obtain the highest bandwidth efficiency irrespectively of distance. We can also see that the proposed RRA always has better performance as its fairness counterpart. Also, when the average normalized distance between AP and RN is around 0.4, we can see that the Fairness SA with an asymmetric link tends to become slightly better than the proposed RRA with symmetric links. The performance of ARA is quite predictable since it utilizes only one RN when assisting the transmission. In Fig. 8, in order to see the performance of relay selection, we also compare our proposed scheme with the cases when a fixed number of relays are always used to assist transmission. The distance between AP-RN is varied in order to see that the optimal number of relays may vary in different situations. One can observe that in the considered case, when AP-RN distance is shorter, three is the optimal number and when the distance is getting longer, two relays form the optimal relay set. In the studied relay deployment scenario the number of optimal relays was never greater than three, which is mainly due to the geometry deployment of RNs.

From a power consumption point of view, our proposed algorithm also shows clear gains over other schemes. In Fig. 9, the sum of the power consumption of all RNs is shown. Compared to the Waterfilling scheme, we can see that our proposed scheme with a symmetric two-hop link achieves significant power savings. That is mainly due to the fact that the proposed schemes can cooperatively allocate power among different RNs while obtaining better throughput, whereas Waterfilling scheme can only perform power allocation based

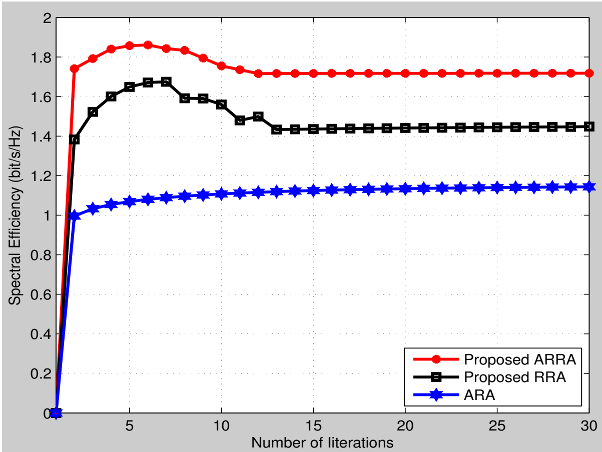

Figure 10. Impact of the number of iteration on the system bandwidth efficiency

on its own channel conditions. Moreover, having the freedom to adjust the time durations between the two hops, better power saving gains can be achieved.

Fig. 10 illustrates the convergence speed of the proposed algorithm and the Fairness SA scheme. We fix the maximum AP power to $P_{s,max} = 40\ dBm$ and maximum RN power to $P_{k,max} = 20\ dBm$. One may notice that at the beginning, due to infeasible solutions, the throughput performance is not stable. After several iterations, the proposed algorithm reaches the steady state, which demonstrates fast convergence speed.

V. Conclusion

In this paper we have investigated the problem of asymmetric resource allocation for collaborative multi-relay OFDMA networks with imperfect channel information. By assuming the knowledge of the statistics of the channel estimation error is available, we proposed an algorithm which can noticeably increase system performance. The joint optimization problem for radio resource allocation was solved by addressing three sub-problems including opportunistic selection of collaborative relays, subcarriers allocation and power allocation with the objective of maximizing the expected system throughput, while allowing the durations between the two hops to differ. It manifested that by designing a proper resource allocation scheme with imperfect CSI for different hops, it is possible to achieve a noticeable gain in the cell-edge throughput.

Appendix A

Derivation of optimal solution in (15) and (16)

For simplicity, we replace $\mathbb{E}_{\gamma_{s,k}^i|\hat{\gamma}_{s,k}^i}[log(1+P_{s,k}^i\gamma_{s,k}^i)]$ with r_1 and $\mathbb{E}_{\gamma_{k,d}^j|\hat{\gamma}_{k,d}^j}\left[log\left(1+\sum_{k=1}^K \rho_k P_{k,d}^j\gamma_{k,d}^j\right)\right]$ with r_2. From (10), we have

$$\frac{T_1}{T_2} = \frac{r_2}{r_1}. \tag{28}$$

Then the derivative of $\mathcal{L}$ in (12) with respect to variable T_1 is given by

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial T_1} &= \frac{(T-T_1)r_1}{T} - \frac{(T_2)r_2}{T} - \mu\Big(\frac{(T-T_1)r_1}{T} - \frac{(T_2)r_2}{T}\Big)\\ &= \Big(\frac{(T-T_1)T_2}{T^2T_1} - \frac{(T_2)}{T} - \mu\Big(\frac{(T-T_1)T_2}{T^2T_1} - \frac{(T_2)}{T}\Big)\Big)r_2.\end{aligned} \tag{29}$$

Since we have $T = T_1 + T_2$ and $\frac{\partial \mathcal{L}}{\partial T_1} = 0$, the (29) can be converted to:

$$\frac{T_2}{T_1} = \frac{1+\mu}{1-\mu}, \tag{30}$$

and we have

$$T_1 = \frac{1-\mu}{2}T, \tag{31}$$

$$T_2 = \frac{1+\mu}{2}T. \tag{32}$$

Appendix B

Derivation of optimal solution in (17) and (18)

For simplicity, we replace $P_{s,k}^i$ with P_1 and $P_{k,d}^j$ with $P_{2,k}$. Similarly, we use G_1 and L_1 to replace $G_{s,k}^i$ and $L_{s,k}$,$G_{2,k}$ and $L_{2,k}$ to replace $G_{k,d}^j$ and $L_{k,d}$. We also replace $\mathbb{E}_{\gamma_{s,k}^i|\hat{\gamma}_{s,k}^i}$ and $\mathbb{E}_{\gamma_{k,d}^j|\hat{\gamma}_{k,d}^j}$ with $\mathbb{E}_{G_1|\hat{G}_1}$ and $\mathbb{E}_{G_{2,k}|\hat{G}_{2,k}}$ respectively. First, we solve the power allocation at the transmitter. When the relay selection and subcarrier allocation are done, the derivative of $\mathcal{L}$ in (12) with respect to variable P_1 is given by

$$\frac{\partial \mathcal{L}}{\partial P_1} = (1-\mu)\frac{T_1}{T}\mathbb{E}_{G_1|\hat{G}_1}\Big[\frac{1}{1+\frac{L_1P_1G_1}{\sigma_k^2}}\frac{L_1G_1}{\sigma_k^2}\Big] - \lambda_s. \tag{33}$$

Applying KKT conditions, we obtain

$$\mathbb{E}_{G_1|\hat{G}_1}\Big[\frac{1}{1+\frac{L_1P_1G_1}{\sigma_k^2}}\frac{L_1G_1}{\sigma_k^2}\Big] = \frac{2\lambda_s}{(1-\mu)^2}. \tag{34}$$

We could approximate the PDF in (3) using Gamma distribution that is known to approximate the non-central Chi-squared distribution quite well [26]. We can obtain

$$f(G_1|\hat{G}_1) \approx \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)}\ G_1^{\alpha_1-1}e^{-\beta_1G_1}, \tag{35}$$

where $\alpha_1 = (\eta_1+1)^2/(2\eta_1+1)$ is the Gamma shape parameter with $\eta_1 = \hat{G}_1/\sigma_h^2$ and $\beta = \alpha_1/(\hat{G}_1+\sigma_h^2)$ is Gamma PDF rate parameter. Hence, by using (35), we simplify the expectation in (34) as follows

$$\begin{aligned}(34) &= \int_0^\infty \frac{1}{1+\frac{L_1P_1G_1}{\sigma_k^2}}\frac{L_1G_1}{\sigma_k^2}f(G_1|\hat{G}_1)dG_1\\ &= \int_0^\infty \frac{1}{1+\frac{L_1P_1G_1}{\sigma_k^2}}\frac{L_1G_1}{\sigma_k^2}\frac{\beta_1^{\alpha_1}}{\Gamma_1(\alpha_1)}\ G_1^{\alpha_1-1}e^{-\beta_1G_1}dG_1\\ &\approx \frac{\beta_1^{\alpha_1}}{P_1\Gamma(\alpha_1)}\int_0^\infty \frac{G_1^{\alpha_1}}{\sigma_k^2/L_1P_1+G_1}e^{-\beta_1G_1}dG_1\\ &= \frac{\alpha_1}{P_1}\Big(\frac{\sigma_k^2\beta_1}{L_1P_1}\Big)^{\alpha_1}e^{\frac{\sigma_k^2\beta_1}{L_1P_1}}\Gamma\Big(-\alpha_1, \frac{\sigma_k^2\beta_1}{L_1P_1}\Big),\end{aligned} \tag{36}$$

where the closed form of the integral is obtained by using [27, page 348, Section 3.383.10] and $\Gamma(a,b)=\int_b^\infty e^{-t}t^{a-1}dt$ is the incomplete Gamma Function. Therefore, using (36) in (34), we could arrive at the approximation of power allocation of the first hop numerically. Similarly, for the second hop, using KKT conditions we arrive at

$$\mathbb{E}_{G_{2,k}|\hat{G}_{2,k}}\Big[\frac{1}{1+\frac{\sum_k^K L_{2,k}P_{2,k}G_{2,k}}{\sigma_d^2}}\frac{L_{2,k}G_{2,k}}{\sigma_d^2}\Big]=\frac{2\lambda_{k,d}}{(1+\mu)^2}. \tag{37}$$

We can see that for $G_{k,d}$, the PDF can be expressed as

$$f(G_{2,k}|\hat{G}_{2,k})\approx\frac{\beta_{2,k}^{\alpha_{2,k}}}{\Gamma(\alpha_{2,k})}G_{2,k}^{\alpha_{2,k}-1}e^{-\beta_{2,k}G_{2,k}}, \tag{38}$$

where $\alpha_{2,k}=(\eta_{2,k}+1)^2/(2\eta_{2,k}+1)$ with $\eta_{2,k}=\hat{G}_{2,k}/\sigma_{\tilde{h}}^2$ and $\beta_{2,k}=\alpha_{2,k}/(\hat{G}_{2,k}+\sigma_{\tilde{h}}^2)$ is Gamma PDF rate parameter. Hence, by using (38), we obtain the expectation in (37)

$$\begin{aligned}(37)=&\int_0^\infty\frac{1}{1+\frac{\sum_k^K L_{2,k}P_{2,k}G_{2,k}}{\sigma_d^2}}\frac{L_{2,k}G_{2,k}}{\sigma_d^2}f(G_{2,k}|\hat{G}_{2,k})dG_{2,k}\\ =&\int_0^\infty\frac{L_{2,k}G_{2,k}}{\sigma_d^2+\sum_k^K L_{2,k}P_{2,k}G_{2,k}}\frac{\beta_{2,k}^{\alpha_{2,k}}}{\Gamma_{2,k}(\alpha_{2,k})}G_{2,k}^{\alpha_{2,k}-1}\\ &e^{-\beta_{2,k}G_{2,k}}dG_{2,k}\\ \approx&\frac{\alpha_{2,k}}{P_{2,k}}\big(c\beta_{2,k}\big)^{\alpha_{2,k}}e^{c\beta_{2,k}}\Gamma\big(-\alpha_{2,k},c\beta_{2,k}\big),\end{aligned} \tag{39}$$

where we have $c=\frac{\sigma_d^2+\sum_{m=1,m\neq k}^K P_{2,m}L_{2,m}G_{2,m}}{L_{2,m}P_{2,m}}$. Then we could obtain approximation of power allocation $P_{2,k}$ by substituting (39) into (37).

Appendix C

Derivation of optimal solution in (19) and (20)

For simplicity, we replace $P_{s,k}^i$ with P_1 and $P_{k,d}^j$ with $P_{2,k}$. Similarly, we use G_1 and L_1 to replace $G_{s,k}^i$ and $L_{s,k}$,$G_{2,k}$ and $L_{2,k}$ to replace $G_{k,d}^j$ and $L_{k,d}$. We also replace $\mathbb{E}_{\gamma_{s,k}^i|\hat{\gamma}_{s,k}^i}$ and $\mathbb{E}_{\gamma_{k,d}^j|\hat{\gamma}_{k,d}^j}$ with $\mathbb{E}_{G_1|\hat{G}_1}$ and $\mathbb{E}_{G_{2,k}|\hat{G}_{2,k}}$ respectively. First, we solve the power allocation at the transmitter. When relay selection and subcarrier allocation are done, the derivative of $\mathcal{L}$ in (12) with respect to variable P_1 is given by

$$\frac{\partial\mathcal{L}}{\partial P_1}=\big(1-\mu\big)\mathbb{E}_{G_1|\hat{G}_1}\big[\frac{1}{1+\frac{L_1P_1G_1}{\sigma_k^2}}\frac{L_1G_1}{\sigma_k^2}\big]-\lambda_s. \tag{40}$$

Applying KKT conditions, we obtain

$$\mathbb{E}_{G_1|\hat{G}_1}\Big[\frac{1}{1+\frac{L_1P_1G_1}{\sigma_k^2}}\frac{L_1G_1}{\sigma_k^2}\Big]=\frac{\lambda_s}{1-\mu}. \tag{41}$$

We could approximate the PDF in (3) using Gamma distribution that is known to approximate the non-central Chi-squared distribution quite well [26]. We can obtain

$$f(G_1|\hat{G}_1)\approx\frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)}\ G_1^{\alpha_1-1}e^{-\beta_1G_1}, \tag{42}$$

where $\alpha_1=(\eta_1+1)^2/(2\eta_1+1)$ is the Gamma shape parameter with $\eta_1=\hat{G}_1/\sigma_{\tilde{h}}^2$ and $\beta_1=\alpha_1/(\hat{G}_1+\sigma_{\tilde{h}}^2)$ is Gamma PDF rate parameter. Hence, by using (42), we simplify the expectation in (41) as follows:

$$\begin{aligned}(41)=&\int_0^\infty\frac{1}{1+\frac{L_1P_1G_1}{\sigma_k^2}}\frac{L_1G_1}{\sigma_k^2}f(G_1|\hat{G}_1)dG_1\\ =&\int_0^\infty\frac{1}{1+\frac{L_1P_1G_1}{\sigma_k^2}}\frac{L_1G_1}{\sigma_k^2}\frac{\beta_1^{\alpha_1}}{\Gamma_1(\alpha_1)}\ G_1^{\alpha_1-1}e^{-\beta_1G_1}dG_1\\ \approx&\frac{\beta_1^{\alpha_1}}{P_1\Gamma(\alpha_1)}\int_0^\infty\frac{G_1^{\alpha_1}}{\sigma_k^2/L_1P_1+G_1}e^{-\beta_1G_1}dG_1\\ =&\frac{\alpha_1}{P_1}\big(\frac{\sigma_k^2\beta_1}{L_1P_1}\big)^{\alpha_1}e^{\frac{\sigma_k^2\beta_1}{L_1P_1}}\Gamma\big(-\alpha_1,\frac{\sigma_k^2\beta_1}{L_1P_1}\big),\end{aligned} \tag{43}$$

where the closed form of the integral is obtained by using [27, page 348, Section 3.383.10] and $\Gamma(a,b)=\int_b^\infty e^{-t}t^{a-1}dt$ is the incomplete Gamma Function. Therefore, using (43) in (41), we could arrive the approximation of power allocation of the first hop numerically. Similarly, for the second hop, using KKT conditions we arrive at

$$\mathbb{E}_{G_{2,k}|\hat{G}_{2,k}}\Big[\frac{1}{1+\frac{\sum_k^K L_{2,k}P_{2,k}G_{2,k}}{\sigma_d^2}}\frac{L_{2,k}G_{2,k}}{\sigma_d^2}\Big]=\frac{\lambda_{k,d}}{1+\mu}. \tag{44}$$

We can see that for $G_{k,d}$, the PDF can be expressed as

$$f(G_{2,k}|\hat{G}_{2,k})\approx\frac{\beta_{2,k}^{\alpha_{2,k}}}{\Gamma(\alpha_{2,k})}G_{2,k}^{\alpha_{2,k}-1}e^{-\beta_{2,k}G_{2,k}}, \tag{45}$$

where $\alpha_{2,k}=(\eta_{2,k}+1)^2/(2\eta_{2,k}+1)$ with $\eta_{2,k}=\hat{G}_{2,k}/\sigma_{\tilde{h}}^2$ and $\beta_{2,k}=\alpha_{2,k}/(\hat{G}_{2,k}+\sigma_{\tilde{h}}^2)$ is Gamma PDF rate parameter. Hence, by using (45), we obtain the expectation in (44)

$$\begin{aligned}(44)=&\int_0^\infty\frac{1}{1+\frac{\sum_k^K L_{2,k}P_{2,k}G_{2,k}}{\sigma_d^2}}\frac{L_{2,k}G_{2,k}}{\sigma_d^2}f(G_{2,k}|\hat{G}_{2,k})dG_{2,k}\\ =&\int_0^\infty\frac{L_{2,k}G_{2,k}}{\sigma_d^2+\sum_k^K L_{2,k}P_{2,k}G_{2,k}}\frac{\beta_{2,k}^{\alpha_{2,k}}}{\Gamma_{2,k}(\alpha_{2,k})}G_{2,k}^{\alpha_{2,k}-1}\\ &e^{-\beta_{2,k}G_{2,k}}dG_{2,k}\\ \approx&\frac{\alpha_{2,k}}{P_{2,k}}\big(c\beta_{2,k}\big)^{\alpha_{2,k}}e^{c\beta_{2,k}}\Gamma\big(-\alpha_{2,k},c\beta_{2,k}\big),\end{aligned} \tag{46}$$

where we have $c=\frac{\sigma_d^2+\sum_{m=1,m\neq k}^K P_{2,m}L_{2,m}G_{2,m}}{L_{2,m}P_{2,m}}$. Then we could obtain approximation of power allocation $P_{2,k}$ by substituting (46) into (44).

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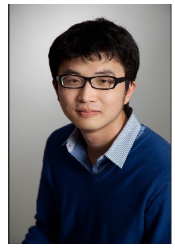

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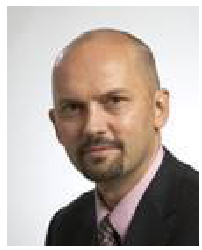

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