OFDM INTERFERENCE ANALYSIS WITH DIRTY RF

Zheng Chang*, Natalia Ermolova†, Olav Tirkkonen†, Tapani Ristaniemi*

* Department of Mathematical Information Technology, University of Jyväskylä, P.O.Box 35, FIN-40014, Jyväskylä, Finland
† Department of Communications and Networking, Aalto University, P.O.Box 13000, FIN-00076 Aalto, Finland

*{zheng.chang, tapani.ristaniemi}@jyu.fi, †{natalia.ermolova, olav.tirkkonen}@tkk.fi

Keywords: dirty RF, OFDM, I/Q imbalance, phase noise, power amplifier

Abstract: When passing through the RF analog electronics devices of the transceiver, the information signal is subject to various distortions named ‘dirty RF’, which degrades the communication system performance. In this paper, we first overview dirty RF effects and then propose an analytical evaluation of the performance of OFDM system where I/Q imbalance, HPA, and phase noise effects are jointly taken into account. We also give an overview of the existed algorithms which can compensate for each RF impairments. Different parameters of these impairments are applied in order to analyze how these RF impairments affect the whole system performance.

1 INTRODUCTION

The future wireless communication systems are expected to provide higher data rates in order to fulfill the requirements of people in the modern society. The limited frequency bandwidth, which often can be seen as an obstacle of the telecommunication development, is also the propulsion of the evaluation of the wireless technology.

The multi-carrier technology, in particularly orthogonal frequency division multiplexing (OFDM), is an effective technique for combating channel noise, multipath effects and enabling high data rate transmissions over fading channels. OFDM has been implemented in many wired and wireless communication systems, such as Long Term Evaluation (LTE) and Worldwide Interoperability for Microwave Access (WiMAX). However, considering the real practical OFDM system, when passing through the RF analog electronics devices of the transceiver, the information signal is subject to various distortions named ‘dirty RF’ effects (Fettweis and Lohning, 2005). Some impairments that have major impacts on the system performance are:

1. Nonlinear high power amplifier: the PA in the reality behaves nonlinearity with respect to signals with amplitude variations, which causes the spectral spreading of the OFDM signal, intermodulation effects of the subcarriers and warping of the signal constellation.

2. Phase noise: phase noise is a random process caused by fluctuation of local oscillators. Phase noise destroys the orthogonalities among the subcarriers, causes constellation rotation and intercarrier interference (ICI).

3. In-phase and quadrature (I/Q) imbalances: In practical devices, the amplitude gain and the phase gain of I and Q branch are never the same. This leads to an attenuation of the system performance.

In this paper, we review some of the most important impairments limiting the performance of OFDM communication system namely: phase noise, I/Q imbalance, non-linearity of high power amplifier. Dirty RF impairments and mitigation techniques have been analyzed mainly separately in the past, so we also joint analyze the system performance of the OFDM system with all these RF impairments.

The rest of this paper is organized as follows: we first overview these three RF impairments in Section II. System model and performance Analysis of joint effects of dirty RF on OFDM system is discussed in Section III. Both theoretical expressions
and simulation results are presented and discussed. Finally, we conclude the paper in Section IV and give future directions for the work.

2 OVERVIEW OF RF IMPAIRMENTS

In this work, some of the most important RF impairments which limit the performance of the OFDM system, such as I/Q imbalance, nonlinearity of power amplifier and phase noise, are presented individually.

2.1 Phase Noise

Phase noise is introduced by the local oscillators (LOs) at both transmitter and receiver. The performance of an OFDM system can be strongly degraded by the presence of random phase noise (Fettweis and Lohning, 2005). It can be described as two multiplicative distortions. However, for a small phase noise bandwidth, the distortion effect approximately equals to the phase noise effect of sum bandwidth of both processes (Petrovic, 2007).

The discrete-time OFDM symbol with phase noise can be expressed as

\[ r(n) = (x(n) \odot h(n))e^{j\Phi(n)} + w(n), \]

where \( x(n) \) and \( w(n) \) denote the samples of the transmitted signal, channel impulse response and the channel noise term respectively, and \( \odot \) denotes convolution. \( \Phi(n) \) represents the phase noise process at the receiver, and is usually modelled as a Wiener process (Armada, 2001).

After taking the discrete Fourier transform (DFT) on the received useful symbols, the demodulated carrier \( R_k \) is:

\[
R_k = X_k H_k I_0 + \sum_{l=0,l\neq k}^{N-1} X_l H_l I_{k-l} + W_k, k = -\frac{N}{2} \ldots \frac{N}{2} - 1
\]

(1)

where \( X_k, H_k \) and \( W_k \) represent the transmitted symbol on the \( k \)th carrier, sampled channel transfer function and frequency domain noise. The term \( I_k \) is

\[
I_k = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\Phi(n)} e^{(-j\frac{2\pi}{N} kn)}
\]

(2)

The term \( I_0 \) in the first term of the Eq. (1), which stems from the phase noise, and does not depend on the subcarrier index, is referred as common phase error (CPE). In the OFDM symbol, for a small phase noise, CPE is (Armada, 2001)

\[
I_0 \approx e^{j\Phi} = 1 + j\Phi
\]

(3)

where the angle \( \Phi \) is

\[
\Phi = \frac{1}{N} \sum_{n=0}^{N-1} \Phi(n).
\]

It can be seen from Eq. (3), CPE results from complex numbers \( e^{j\Phi(n)} \). Therefore, it can be viewed as a rotation on the signal constellation. The second term of the Eq. (1) is ICI, corresponding to the summation of the subcarrier each multiplied by a complex number. The spectral component of phase noise in this error term is randomized, therefore, it can not be corrected totally (Armada, 2001).

Phase noise effects on OFDM system have been analyzed over a period. Several compensation methods based on estimation and correction have been introduced i.e. in (Petrovic, 2007), and (Armada, 2001). In (Petrovic, 2007), an ICI suppression algorithm is presented without considering CPE. The proposed algorithm provides estimation of as many spectral components \( \hat{I}_k \) as possible. Then the estimates \( \hat{I}_k \) are used to do the ICI cancellation. (Armada, 2001) focuses only on correction of CPE and takes advantage of the pilot-based correction mechanism.

2.2 I/Q Imbalance

Another major source of impairments in wireless communication system is a mismatch between the I and Q branches or, equivalently between the real and imaginary parts of the complex signal at both transmitter(during up-conversion) and receiver(during down-conversion). Ideally, I and Q branches should be matched perfectly, which never happens in reality. It results in the so called I/Q imbalance.

At the transmitter, since I/Q imbalance is any mismatch between the I and Q branches from the ideal case, the distorted signal in the time domain can be modelled as (Tarighat, 2007):

\[
x_d(t) = \mu_t x(t) + \nu_t x^*(t)
\]

(4)

where \( \mu_t = \cos(\Delta \phi_t) + j\nu_t \sin(\Delta \phi_t) \) and \( \nu_t = \gamma_t \cos(\Delta \phi_t) + j\sin(\Delta \phi_t) \) are related to the amplitude imbalance \( \gamma_t \) and phase imbalance \( \Delta \phi_t \) between the I and Q branches at the transmitter. The notation * denotes the complex conjugate.

It can be seen that if \( \gamma_t = \Delta \phi_t = 0 \), then, \( \mu_t = 1, \nu_t = 0 \) and \( x_d(t) = x(t) \), i.e., there is no I/Q imbalance.

At the receiver, the received signal \( R \) can be expressed as:
\[ R = \mu_r X_H + \nu_r X_m^2 H + W \]  
\[ l_m = \begin{cases} 
2 + N - 1, & l = 2, \ldots, N \\
1, & l = 1 
\end{cases} \]  

\[ R_d = \mu_d + R + \nu_d^2 \]  

From the Eq. (5), (7) and (8), it can be seen that I/Q imbalance can cause the ICI at the receiver. Since OFDM system is sensitive to ICI, I/Q imbalance may cause severe performance degradation.

Different compensation algorithms for I/Q imbalance has been investigated (Tubbax and Come, 2004) focuses on the case where the I/Q imbalance is only observed at the OFDM transmitter. A MMSE estimation algorithm is proposed to estimate the \( \mu_i \) and \( \nu_i \), then the compensation method is based on the estimated value of \( \mu_i \) and \( \nu_i \). (Tarighat and Baghari, 2005) presents several different compensation schemes for I/Q imbalance at OFDM receiver, including least squares (LS) channel estimation and equalization, least mean squares (LMS) equalization and distortion estimation. More details about compensation algorithms of I/Q imbalance are in (Chang, 2009).

### 2.3 Nonlinearity of Power Amplifier

PA is an essential component of the transmitter in a modern wireless communication system. Ideally, the PA should be linear. However, in reality all the PAs have nonlinear input-output characteristics which generates nonlinear distortion with respect to the signals with envelope fluctuations (Cripps, 2002). When the input power remains at a low level, the output power of the PA is approximately linear with respect to the input power. As the input power increases, a PA AM/AM curve becomes more nonlinear. Finally when the input power does not drive the change of the output power, the amplifier reaches its saturation point.

Some parameters are used to describe how efficiently the amplifier is used. The one that is used in this work is output back-off (OBO). OBO is the difference between the output saturation power and output average power, which is defined as:

\[ OBO = 10 \log_{10} \frac{P_{out\text{sat}}}{P_{out\text{ave}}} \]  

where \( P_{in\text{sat}} \) and \( P_{out\text{sat}} \) stand for the input and output saturation power respectively. The output-input relation of the memoryless nonlinear PA can be described as (Cripps, 2002):

\[ u(t) = F(x(t)) \]  

where \( F(\cdot) \) is a nonlinear function. The output of the nonlinear power amplifier can be written as:

\[ u(t) = F_A(\rho(t))e^{jF_P(\rho(t))}e^{j\psi(t)} \]  

where \( F_A(\cdot) \) and \( F_P(\cdot) \) are the AM-AM and AM-PM conversion functions of the amplifier.

There exists a number of linearization techniques, which can be divided in a few groups: Feedback, Feedforward, Predistortion and others (Cripps, 2002). More details about compensation algorithms are in (Chang, 2009).

### 3 SYSTEM MODEL AND PERFORMANCE ANALYSIS

#### 3.1 System Model of Joint Effect

An implementation of dirty RF at OFDM transceiver is described as follows. At the transmitter, I/Q imbalance is considered. The nonlinear effect of the PA can also generates signal distortion. At the receiver, the receiver LO cause both I/Q imbalance and phase noise.

The OFDM signal with I/Q imbalance can be written as a function of \( \mu_i, \nu_i \)

\[ x_{iq}(t) = \mu_i x(t) + \nu_i x^*(t) \]  

The imbalanced signal \( x_{iq}(t) \) is then amplified by a nonlinear PA. The output of the nonlinear PA also can be modeled as:

\[ x_{d-\text{iq}}(t) = Kx_{iq}(t) + d(t) \]  

\( K \) can be viewed as a constant number for different certain value of OBO (Costa and Pupolin, 2002). In
such case, \( d(t) \) is additive Gaussian-like noise with zero mean with variance \( \sigma_d^2 \).

In case of phase noise at the receiver, the received signal can be expressed as:

\[
    r_{pm}(t) = r(t)e^{j\phi(t)}
\]

where \( r(t) \) is the received signal, \( \phi(t) \) represents the phase noise process at the receiver, and it is modelled by Wiener process here.

The impact of I/Q imbalance on the received signal with phase noise can be modelled as:

\[
    r_{iq-pm}(t) = \mu_r r_{pm}(t) + v_{r-pm} e^{j\phi(t)}
\]

Therefore, by using Eq. (12) and (14), the signal with I/Q imbalance and phase noise can be written as:

\[
    r(t) = h(t) \otimes x_{d-iq}(t) + w(t)
\]

where \( w(t) \) is the channel noise. After several manipulations, and for simplicity, we do not use \( t \), the received signal with all RF impairments can be obtained as:

\[
    r_{pa-iq-pm} = (a_1 e^{j\theta} + a_2 e^{-j\theta}) h \otimes x +
    (b_1 e^{j\theta} + b_2 e^{-j\theta}) h^* \otimes x^* +
    (\mu_r v_{r-pm} e^{j\phi} + v_{r-pm} e^{-j\phi})
\]

where \( v = d + w, a_1 = K_{\mu_r}\mu_r, a_2 = K' v_r, b_1 = K_{\mu_r} v_r, b_2 = K' \mu_r v_r \). After OFDM demodulation, the resulting frequency domain signal is given by:

\[
    R = (a_1 - a_2) X H I(0) + (a_1 - a_2) I C I_b +
    (b_1 - b_2) X^*_m H^*_a I(0) + (b_1 - b_2)(I C I_b)_m^* + \xi
\]  

In Eq. (19), all the RF impairments have been considered in the OFDM baseband signal. It is clear that PA nonlinearity introduces distortion factor \( K \) and \( d(n) \) is the I/Q imbalance at both transmitter and receiver bring \( h_i, \mu_r, v_i \) and \( v_r \) to the subcarrier signal and the mirror frequency subcarrier signal. I/Q imbalance causes ICI because the power leaks from the signal on the mirror frequency subcarriers are under consideration. Meanwhile, the phase noise effect introduces a CPE when \( I(0) = e^{j\theta} \) as well as ICI (Tsai and Liao, 2008).

3.2 Simulation Results and Performance Analysis

The modulation method is M-QAM and number of subcarrier is \( N \) which is exponent of 2. One frequently used PA named solid-state power amplifier (SSPA) is considered in this work. The OBO value is changed by turning the input power and the saturation level is constant. The channel is modelled as both AWGN channel and Rayleigh fading channel.

Fig.1a shows the BER performance of the OFDM system with dirty RF, including I/Q imbalance, phase noise and nonlinear PA. The parameters are \( \Delta_\phi = \gamma_\phi = 5^\circ, \gamma_r = 0.05 \) for I/Q imbalance, variance of the phase noise is 0.04. SSPA model is used with OBO = 4.2 dB for 4-QAM, OBO = 3.8 dB for 16-QAM and OBO = 4.0 dB for 64-QAM. As shown in the Fig.1, dirty RF effects severely degrade the performance even in the 4-QAM case. For 16-QAM and 64-QAM, the BER performances are totally unacceptable. The same performance can be found in Fig.1b, in which a Rayleigh fading channel is considered.

Next, we change parameters of one RF impairment to see how individual RF impairment affects the system when considering whole dirty RF effects. Fig.2a shows BER performance of the system with different parameters of I/Q imbalance. We consider the same I/Q imbalance for both transmitter and receiver. The modulation scheme is 4-QAM. The variance of phase noise is 0.04 and SSPA is used with OBO = 4.3 dB.

We can see from Fig.2a, when the \( \Delta_\phi = \Delta_\phi_r < 5^\circ, \gamma_r = 0.05 \), the degradation is not so strong, i.e. for BER=10^{-3}, the degradation is slightly more than 3 dB. However, when \( \Delta_\phi = \Delta_\phi_r = 10^\circ, \gamma_r = 0.10 \), the degradation is much worse than others, i.e. when BER = 10^{-2} in Fig.2a, the degradation is more than 7 dB comparing to no I/Q imbalance sys-
tem. In Fig. 2b, we can see that the degradation when $\Delta \varphi_t = \Delta \varphi_r < 5^\circ, \gamma = \gamma_r < 0.05$ is even smaller that the one in AWGN case. If the 0.01-0.02 amplitude imbalance and $1^\circ - 2^\circ$ phase imbalance is realistic, from Fig.2a, the degradation caused by I/Q imbalance can be thought as very slight.

If we fix the parameters of I/Q imbalance and non-linear PA to be constant and change the variance of the phase noise, we could see the effects of phase noise in the joint effects. The parameters are $\Delta \varphi_t = \Delta \varphi_r = 5^\circ, \gamma = \gamma_r = 0.05$ for I/Q imbalance, and SSPA model is used with OBO = 4.2 dB for 4-QAM and OBO = 7.4 dB for 16-QAM. The BER performances of OFDM system with different variances of phase noise in AWGN channel are presented in the Fig.3a and b.

![Figure 2: BER performance of OFDM with joint effects, different I/Q imbalance parameters, 4-QAM, AWGN(a) and Rayleigh fading channel(b)](image)

a. variance = 0  b. variance = 0.04  c. variance = 0.16  d. variance = 0.6

![Figure 4: Frequency domain histograms of interferences caused by joint effects, different variance of phase noise](image)

Different curves in each figure correspond to different values of variance of phase noise. It is apparent that the performance degrades strongly for variance $> 0.04$ in Fig.3a. For the case of 16-QAM in Fig.3b, we can see that even without phase noise and with a high OBO value, the BER performance is strongly degraded by the other two impairments. The BER curve for variance = 0.01 and the curve for no phase noise are nearly same, which stand for that when variance is very small, the impairment for the system performance is slight. Comparing to 4-QAM, there is no such gap between BER curves of different variance.

Fig. 5a,b and c show the frequency domain PDF histograms of the interference caused by joint effects when different variances of phase noise are used. From these four figures, we could see when the variance is small, the PDF still follow the Gaussian distribution, however, as the value of variance increases, the PDF is not following Gaussian very well, i.e. Fig.5d.

![Figure 5: Frequency domain histograms of interferences caused by joint effects, different variance of phase noise](image)
4 CONCLUSION AND FUTURE

In this paper, three dirty RF impairments that degrade the performance of OFDM system are introduced in details. The models of phase noise, I/Q imbalance as well as nonlinearity of PA are presented. Simulation results can demonstrate how these impairments limit the performance of systems. Following discussions are made to illustrate the joint effects.

One important research direction is to derive a compensation algorithm for the joint effects. One possible way to do compensation is to combine the regular algorithms for individual RF impairments together. However, considering the complexity of the algorithm and techniques for mitigation of one factor may enhance the impact of another one, it is better to find more suitable way to compensate all the RF impairments together.

ACKNOWLEDGEMENTS

The author would like to thank Finnish Funding Agency for Technology and Innovation (Tekes) for funding.

REFERENCES