Resource Allocation for Cooperative Relay-assisted OFDMA Networks with Imperfect CSI

Zheng Chang*† and Tapani Ristaniemi*
*Department of Mathematical Information Technology, University of Jyväskylä, P.O.Box 35, FIN-40014 Jyväskylä, Finland
† Magister Solutions Ltd, Hannikaisenkatu 41, FIN-40100 Jyväskylä, Finland
Email: {zheng.chang, tapani.ristaniemi}@jyu.fi

***Abstract*—This work addresses the radio resource allocation (RRA) problem for cooperative relay assisted OFDMA wireless network. The relays adopt the decode-and-forward protocol and can cooperatively assist the transmission from source to destination. The RRA scheme addresses practical implementation issues of resource allocation in OFDMA networks: the inaccuracy of channel-state information (CSI) available to the source. Instead, the source only knows estimated channel status and distributions of related estimation errors. The objective is to maximize the system throughput of the source-to-destination link under various constraints. Since the optimization problem is known as NP-hard, we divide the original problem to three subproblems including relay selection, subcarrier and power allocations. We derive theoretical expressions for the solutions and illustrate them through simulations. Results validate clearly that our proposed algorithm can enhance the performance of system with imperfect CSI compared to the other newly proposed resource allocation schemes.**

***Index Terms*—OFDMA, relay selection, subcarrier allocation,power allocation, imperfect CSI and cooperative communications.**

I. Introduction

Orthogonal Frequency Division Multiple Access (OFDMA) is an effective technique that exploits the benefits of Orthogonal Frequency Division Multiplexing (OFDM) for combating against channel noise and multipath effects and finally enables high data rate transmissions over fading channels. Meanwhile, cooperative communication has emerged as one of the main trends to reach even better system performance in terms of throughput, energy efficiency or cell coverage. Therefore, the incorporation of OFDMA and cooperative relays is foreseen to result in a promising structure that offers a possibility to reach many desirable objectives for the future wireless networks. However, a combination of a conventional one-to-many (single hop) OFDMA system and a relay network calls for a careful design of the radio resource allocation (RRA) principles. This means a carefully design and coordination of the power and subcarrier allocation, selection of relay(s) across different hops and optimizing the resource between the hops.

The RRA algorithm plays an important role in the developments of both conventional and relay-aided OFDMA systems. The related works have been widely done in several different areas [1]-[6] when assuming perfect channel state information (CSI) is known to the source. A cross-layer optimization algorithm for resource allocation in conventional OFDMA network has been presented in [1] without considering relaying. An iterative algorithm is proposed to solve the subcarrier assignment together with relay selection in [2]. Then, the power allocation problem can be solved by another iterative method based on waterfilling algorithm. Authors in [3] introduces closed-form solution for radio resource allocation for multihop cooperative relay network. However, the per-tone power constraint is used which is not practical. Scheme used in [4] considers fairness constraints when selecting relays. In [5], a threshold method is used to solve two subproblems, subcarrier allocation and power allocation. Although the performance is improved comparing to some other algorithms, the total power constraint is considered, which is not a realistic case since each node has its own power limitation. The work in [6] also proposed a subcarrier and relay pairing algorithm to solve the existing RRA problem, which requires high complexity. [7]-[9] present the work about RRA with imperfect CSI. [7] consider the RRA algorithm for conventional OFDMA networks. [8] investigates the issue of joint RRA and relay selection with imperfect CSI. However, authors use mean rate to characterize the CSI uncertainty which results in different interpretations. Recent work about RRA for OFDMA relay networks with imperfect CSI is introduced in [9], where only one relay is selected for assisting the transmission.

As we can see, the existed algorithms have their major drawbacks which need to be improved. In this paper, we investigate a new resource allocation scheme for OFDMA cooperative network with imperfect CSI, which can effectively solve the problems of joint relay selection, subcarrier and power allocation and thus, enhance the system throughput when estimated error existed. In this work, relays are deployed for extending the cell coverage, so we do not consider direct link from source to destination. Conditional expected throughput is considered as our performance evaluation metric. We propose a relay selection and subcarrier allocation schemes, where one set of relays that can obtain the best link data rate is selected. Power is allocated to the source and relays under per-node constraints, which is more realistic than the scheme , e.g., in [5] where only whole system power summit is considered. To the best of our knowledge, such joint optimization for assumed two hop OFDMA network with imperfect CSI has not been studied in the literatures and is important for achieving the better overall system performance.

The rest of this paper is organized as follows. Section II describes our relay-assisted OFDMA cooperative wireless networks and formulates the problem. We consider downlink only in this work, but it can be extended further to the uplink case. In Section III, the proposed resource allocation scheme is presented. We demonstrate the benefits of our proposed algorithm in section IV and finally conclude the paper in Section V.

II. System Model and Problem Formulation

A. System model and Assumptions

We consider our system as a two-hop downlink OFDMA relay network. The whole system consists of source(i.e. access point, AP), destination node(i.e. mobile terminal, MT) and several relays. The first hop is so called broadcast phase, where AP broadcasts information to a cluster of decode-and-forward (DF) relays. In the second hop, relays cooperate to transmit the information data to the MT, so that, e.g., spatial diversity gain can be achieved (relays are assumed to be far enough to each other). The estimated CSI is assumed to be known at MT through estimator (e.g., minimum mean square error estimator (MMSE)) and then feed back to the transmitter perfectly. We also assume that channel estimation error pertains to the amplitude of the correct channel gain, while the phase of the channel gain can be perfectly obtained. As a result, estimated channel gain with an estimation error is available to both the transmitter and the receiver. The AP acts as a central controller to carry out all resource allocation related operations based on the feedbacks from the MT.

In this work, we assume there are total Z relays in the networks, and the selected relay cluster $\mathcal{K}$ contains K potential half-duplex relays. The presented relay-assisted cooperative OFDMA network is as shown in Fig. 1 when $K = 3$.

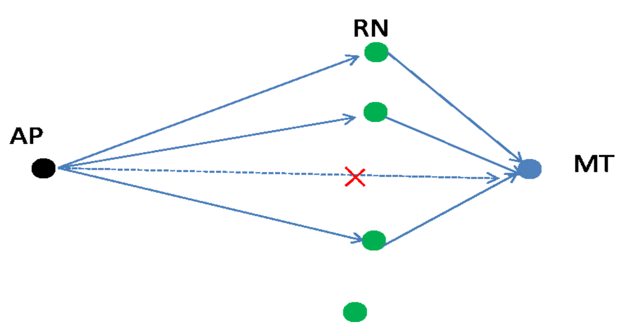

Figure 1. Wireless cooperative relay networks

B. Problem Formulation

Let x be the transmit data from transmitter to receiver and P is the transmit power gain. So regardless of the path loss, the received data after estimator at receiver is

$$y = h\sqrt{P}x + n, \tag{1}$$

and we have

$$h = \hat{h} + \tilde{h}, \tag{2}$$

where $\hat{h}$ is the estimated channel function and $\tilde{h}$ is the independent estimation error which can be modeled as zero mean Gaussian random variable with variance $\sigma^2_{\tilde{h}}$. Thus, the imperfect CSI h is assumed to follow $\mathcal{CN}(\hat{h}, \sigma^2_{\tilde{h}})$. n is the additive noise which can be also modeled as complex Gaussian random variable with variance σ^2_n. Therefore, the square of imperfect CSI h follows a noncentral chi-square probability density function (PDF) given by

$$f(G|\hat{G}) = \frac{1}{\sigma^2_{\tilde{h}}} e^{-\frac{\hat{G}+G}{\sigma^2_{\tilde{h}}}} \mathcal{J}_0\Big(2\sqrt{\frac{\hat{G}G}{\sigma^4_{\tilde{h}}}}\Big) \tag{3}$$

where we denote $G = |h|^2$,$\hat{G} = |\hat{h}|^2$. $\mathcal{J}_0$ is the 0th order modified Bessel Function of the first kind.

In our proposed system model, we suppose h^i is the channel transfer function from transmitter to receiver and we assume the channel is static in a time slot. For example, $\hat{h}^i_{s,k}$ means the channel estimate from AP s to relay node (RN) k over OFDM subcarrier i and $\hat{h}^j_{k,d}$ means the channel estimate from RN k to destination d over OFDM subcarrier j. We have channel gain of the first hop $\hat{G}^i_{s,k} = |\hat{h}^i_{s,k}|^2$ and second hop $\hat{G}^j_{k,d} = |\hat{h}^j_{k,d}|^2$. L is the path loss factor and the noise variance for two hops are σ^2_k and σ^2_d. The variance of related estimation error for two hops are $\sigma^2_{\tilde{h},k}$ and $\sigma^2_{\tilde{h},d}$ and we assume $\sigma^2_{\tilde{h}} = \sigma^2_{\tilde{h},k} = \sigma^2_{\tilde{h},d}$. We denote the transmit power assigned to subcarrier i for transmitting data as P^i. In this work, we do not consider the direct link from AP to MT due to distance or obstacles. This assumption is practical for the case that RNs are deployed for cell extension. One RN k occupies subcarrier i in the first hop and j in the second hop. Therefore, at first hop, the data rate of the broadcast phase is determined by the minimum rate of each link between AP and selected RNs. Since transmitter only knows the CSI conditioned on the feedback of receiver, we could obtain the expected achievable throughput of the first hop as follows:

$$R^{\mathcal{I}}_{s,\mathcal{K}} = \min_{k\in\mathcal{K}} \Big\{ \mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}} \Big[log(1 + \sum_{i=1}^{M} \omega^i_{s,k}\rho_k P^i_{s,k}\gamma^i_{s,k}) \Big] \Big\}, \tag{4}$$

where $\gamma^i_{s,k} = \frac{L_{s,k}G^i_{s,k}}{\sigma^2_k}$ and $\hat{\gamma}^i_{s,k} = \frac{L_{s,k}\hat{G}^i_{s,k}}{\sigma^2_k}$. The notation $\mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}}$ means expectation with respect to $\gamma_{s,k}$ conditioned on $\hat{\gamma}^i_{s,k}$. $\mathcal{M}$ is the subcarrier set of the system that contains M subcarriers. $\mathcal{I}$ is the subcarrier set which contains the subcarriers that are allocated to the selected RNs at first hop. We further refer the link throughput and its lower bound interchangeably for simplicity. ρ_k indicates that whether RN k is chosen for subcarrier allocation, so we obtain

$$\rho_k = \begin{cases} 1 & \text{if } k \text{ is chosen for relaying,} \\ 0 & \text{otherwise.} \end{cases}$$

We also define ω is the indicator whether certain subcarrier is assigned to RN k, for example,

$$\omega^i_{s,k} = \begin{cases} 1 & \text{if } i \text{ is assigned to } k \text{ at first hop,} \\ 0 & \text{otherwise.} \end{cases}$$

For the second hop, it is assumed that the RNs are perfectly synchronized and transmitted at the same time. Therefore, the second hop can be viewed as a virtual MISO link. The expected throughput can be expressed as

$$R_{\mathcal{K},d}^{\mathcal{J}} = \mathbb{E}_{\gamma_{k,d}^{j}|\hat{\gamma}_{k,d}^{j}}\Big[log\big(1+\sum_{j=1}^{M}\sum_{k=1}^{K}\omega_{k,d}^{j}\rho_k P_{k,d}^{j}\gamma_{k,d}^{j}\big)\Big], \quad (5)$$

where $\gamma_{k,d}^{j} = \frac{L_{k,d}G_{k,d}^{j}}{\sigma_d^2}$ and $\hat{\gamma}_{k,d}^{j} = \frac{L_{k,d}\hat{G}_{k,d}^{j}}{\sigma_d^2}$. $\mathcal{J}$ is the subcarrier set which contains the subcarriers that are allocated to the selected RNs at second hop. For indicator $\omega_{k,d}^{j}$, we also have

$$\omega_{k,d}^{j} = \begin{cases} 1 & \text{if } j \text{ is assigned to } k \text{ at second hop,} \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, the total achieved end-to-end throughput of source s to destination d through RN set $\mathcal{K}$ is [10]

$$R_{sd} = min\ \frac{1}{2}\Big\{R_{s,\mathcal{K}}^{\mathcal{I}}, R_{\mathcal{K},d}^{\mathcal{J}}\Big\}. \quad (6)$$

Then, we can formulate our problem as

$$max\ \ R_{sd}, \quad (7)$$

subject to

$$\begin{aligned} &\sum_{i=1}^{M}\sum_{k=1}^{K}\omega_{s,k}^{i}P_{s,k}^{i} \le P_{s,max} \\ &\sum_{j=1}^{M}\omega_{k,d}^{j}P_{k,d}^{j} \le P_{k,max} \\ &\sum_{k=1}^{K}\omega_{s,k}^{i} = 1, \omega_{s,k}^{i} \in \{0,1\} \\ &\sum_{k=1}^{K}\omega_{k,d}^{j} = 1, \omega_{k,d}^{j} \in \{0,1\} \end{aligned} \quad (8)$$

where $P_{s,max}$ is the maximum transmit power of AP and $P_{k,max}$ is the maximum power of RN. Therefore, our goal is to find the optimal solutions of relay, subcarrier and power allocations which satisfy the problem (7).

It can be deduced that (6) can achieve maximum only when $R_{s,\mathcal{K}}^{\mathcal{I}} = R_{\mathcal{K},d}^{\mathcal{J}}$. Thus, (7) can be modified to

$$\arg\max\ \Big(R_{s,\mathcal{K}}^{\mathcal{I}} + R_{\mathcal{K},d}^{\mathcal{J}}\Big), \quad (9)$$

subject to conditions in (8) and

$$R_{s,\mathcal{K}}^{\mathcal{I}} = R_{\mathcal{K},d}^{\mathcal{J}}. \quad (10)$$

III. Resource Allocation Scheme

In this section, we introduce the adaptive algorithms to solve existing problem (9). Although the resource allocation problem is combinatorial in nature with nonconvex structure, as the number of subcarriers becomes sufficient large, the dual gap tends to be zero [12]. Therefore, it can be solved in dual domain. The Lagrangian of problem (9) is [11]

$$\begin{aligned} \mathcal{L}(\mathbf{P},\boldsymbol{\omega},\boldsymbol{\rho},\boldsymbol{\lambda},\boldsymbol{\mu}) =& \Big(R_{s,\mathcal{K}}^{\mathcal{I}} + R_{\mathcal{K},d}^{\mathcal{J}}\Big) \\ &-\lambda_s\Big(\sum_{i=1}^{M}\sum_{k=1}^{K}\omega_{s,k}^{i}P_{s,k}^{i} - P_{s,max}\Big) \\ &-\sum_{k=1}^{K}\lambda_{k,d}\Big(\sum_{j=1}^{M}\omega_{k,d}^{j}P_{k,d}^{j} - P_{k,max}\Big) \\ &-\mu(R_{s,\mathcal{K}}^{\mathcal{I}} - R_{\mathcal{K},d}^{\mathcal{J}}), \end{aligned} \quad (11)$$

where $\mathbf{P} = \{P_{s,k}^{i}, P_{k,d}^{j}\}$ is the set of power allocation, $\boldsymbol{\omega} = \{\omega_{s,k}^{i}, \omega_{k,d}^{j}\}$ denotes the subcarrier allocation, and $\boldsymbol{\rho} = \{\rho_k\}$ is the relay assignment. The $\boldsymbol{\lambda} = \{\lambda_s, \lambda_{k,d}\}$ and $\boldsymbol{\mu} = \{\mu\}$ are Lagrange multipliers. Then it can be derived that $\lambda_s, \lambda_{k,d} \ge 0$ and $\mu \in (-1,1)$. The Lagrange dual function can be written as:

$$g(\boldsymbol{\lambda},\boldsymbol{\mu}) = \max \mathcal{L}(\mathbf{P},\boldsymbol{\omega},\boldsymbol{\rho},\boldsymbol{\lambda},\boldsymbol{\mu}). \quad (12)$$

Since we assume the number of subcarrier is sufficient large, so that the duality gap between primal problem and dual function can be negligible [12]. consequently, we can solve the problem (7) by minimizing the dual function

$$\min\ \ g(\boldsymbol{\lambda},\boldsymbol{\mu}). \quad (13)$$

A. Evaluating Dual Variable

Since a dual function is always convex [11], then for example, two methods can be used to minimize $g(\boldsymbol{\lambda},\boldsymbol{\mu})$ with guaranteed convergence, which are subgradient method and ellipsoid method [12].

We follow the subgradient method in [12] to derive the subgradient $g(\boldsymbol{\lambda},\boldsymbol{\mu})$ with the optimal power allocation p^* that will be presented in the following subsection.

Algorithm 1 Evaluating Dual Variable

```
1: Initialize λ^0 and μ^0
2: while (!Convergance) do
3:    Obtain g(λ^a, μ^a) at the a-th iteration;
4:    Update a subgradient for λ^{a+1} and μ^{a+1}, by λ^{a+1} =
      λ^a + υ^a △λ and μ^{a+1} = μ^a + υ^a △μ;
5: end while
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where $\triangle\boldsymbol{\lambda} = \{\triangle\lambda_s, \triangle\lambda_{1,d}, ...\triangle\lambda_{K,d}\}$, $\triangle\lambda_s$, $\triangle\lambda_{k,d}$ and $\triangle\mu$ can be expressed as

$$\begin{aligned} \triangle\lambda_s &= P_{s,max} - \sum_{i=1}^{M}\sum_{k=1}^{K}(P_{s,k}^{i})^* \\ \triangle\lambda_{k,d} &= P_{k,max} - \sum_{j=1}^{M}(P_{k,d}^{j})^* \\ \triangle\mu &= (R_{s,\mathcal{K}}^{\mathcal{I}})^* - (R_{\mathcal{K},d}^{\mathcal{J}})^*. \end{aligned} \quad (14)$$

Here υ^a is the stepsize and a is the number of iterations. The subgradient algorithm in Algorithm 1 is guaranteed to converge to the optimal $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$. The computational complexity of Algorithm 1 is polynomial in the number of dual variable $K+1$ [12]. Since (12) can be viewed as nonlinear integer programming problem, whose optimal solution requires high computational cost. Therefore, we are aiming to solve the optimization problem by solving three subproblems, which are relay selection, subcarriers and power allocation. We firstly introduce power allocation scheme.

B. Power Allocation Scheme

In order to obtain the optimal solution of power allocation, we are aiming to solve the problem solving problem (11) over variables $P^i_{s,k}$ and $P^j_{k,d}$. However, from (5) and (4), we see that problem (11) involves the conditional expectation of achievable throughput with respect to estimated CSI. Applying Karush-Kuhn-Tucker (KKT) conditions [11] and equations in [14], we could obtain the optimal power allocation schemes by solving following equation numerically.

$$\frac{\alpha^i_{s,k}}{P^i_{s,k}}\Big(\frac{\sigma_k^2\beta^i_{s,k}}{L_{s,k}P^i_{s,k}}\Big)^{\alpha^i_{s,k}} e^{\frac{\sigma_k^2\beta^i_{s,k}}{L_{s,k}P^i_{s,k}}}\Gamma\Big(-\alpha^i_{s,k},\frac{\sigma_k^2\beta^i_{s,k}}{L_{s,k}P^i_{s,k}}\Big)=\frac{\lambda_s}{1-\mu}. \tag{15}$$

where $\Gamma(a,b)$ is the incomplete Gamma function. $\alpha^i_{s,k}=(\eta^i_{s,k}+1)^2/(2\eta^i_{s,k}+1)$ is the Gamma shape parameter with $\eta^i_{s,k}=\hat{G}^i_{s,k}/\sigma^2_{\tilde{h}}$ and $\beta^i_{s,k}=\alpha^i_{s,k}/(\hat{G}^i_{s,k}+\sigma^2_{\tilde{h}})$ is Gamma PDF rate parameter. Similarly, for the cooperation phase, the optimal RN power allocation is obtained by solving:

$$\frac{\alpha^j_{k,d}}{P^j_{k,d}}\big(c_1\beta^j_{k,d}\big)^{\alpha^j_{k,d}} e^{c_1\beta^j_{k,d}}\Gamma\big(-\alpha^j_{k,d},c_1\beta^j_{k,d}\big)=\frac{\lambda_{k,d}}{1+\mu}, \tag{16}$$

where $\alpha^j_{k,d}=(\eta^j_{k,d}+1)^2/(2\eta^j_{k,d}+1)$ with $\eta^j_{k,d}=\hat{G}^j_{k,d}/\sigma^2_{\tilde{h}}$ and $\beta^j_{k,d}=\alpha^j_{k,d}/(\hat{G}^j_{k,d}+\sigma^2_{\tilde{h}})$. We have $c_1=\frac{\sigma_d^2+\sum_{m=1,m\neq k}^{K}P_{m,d}L_{m,d}G_{m,d}}{L_{k,d}P^j_{k,d}}$. $P_{m,d}$ and $G_{m,d}$ is the power allocation and channel gain from relay m to MT d. By using approximation method, e.g. , in [15], we are able to obtain the power allocation with imperfect CSI. One example can be found in Fig. 2 where different value of $\sigma^2_{\tilde{h}}$ is considered. We can see that when estimated error is relative small, the power allocation achieved by imperfect CSI is very close to the one when perfect CSI is assumed at AP.

C. Optimal Relay Selection (ORS)

We consider ORS in this work, unlike some traditional single relay selection algorithms in [6] and [13], as the multiple RNs selection. The proposed algorithm is to select K RNs to form a cluster that can maximize the achieved throughput in (6) based on the imperfect CSI.

Assuming the subcarrier and power allocation is done, we can rewrite (11) as

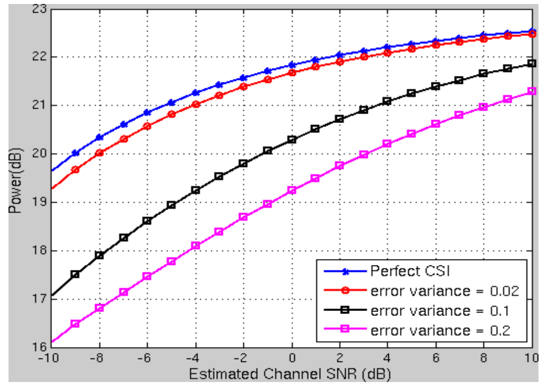

Figure 2. One example of power allocation as function of estimated channel SNR for various value of $\sigma^2_{\tilde{h}}$.

$$\begin{aligned}\mathcal{L}(\mathbf{P},\boldsymbol{\omega},\boldsymbol{\rho},\boldsymbol{\lambda}) &= \min_{k\in\mathcal{K}}\Big\{\mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}}[log(1+\sum_{i=1}^{M}\omega^i_{s,k}\rho_k P^i_{s,k}\gamma^i_{s,k})]\Big\}\\ &+\mathbb{E}_{\gamma^j_{k,d}|\hat{\gamma}^j_{k,d}}[log(1+\sum_{i=1}^{M}\sum_{k=1}^{K}\omega^i_{s,k}\rho_k P^j_{k,d}\gamma^j_{k,d})]\\ &-\mu\Big(\min_{k\in\mathcal{K}}\Big\{\mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}}[log(1+\sum_{i=1}^{M}\omega^i_{s,k}\rho_k P^i_{s,k}\gamma^i_{s,k})]\Big\}\\ &-\mathbb{E}_{\gamma^j_{k,d}|\hat{\gamma}^j_{k,d}}[log(1+\sum_{i=1}^{M}\sum_{k=1}^{K}\omega^i_{s,k}\rho_k P^j_{k,d}\gamma^j_{k,d})]\Big)\\ &+\lambda_s(\sum_{i=1}^{M}\sum_{k=1}^{K}\omega^i_{s,k}P^i_{s,k}-P_{s,max})\\ &-\sum_{k=1}^{K}\lambda_{k,d}(\sum_{j=1}^{M}\omega^j_{k,d}P^j_{k,d}-P_{k,max}).\end{aligned} \tag{17}$$

By applying KKT condition, the RN is selected according to the following rule,

$$\begin{aligned}\mathcal{K}^* = \arg\max_{k} &\Big(\min_{k\in\mathcal{K}}\Big\{(1-\mu^*)\mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}}\Big[\frac{P_{s,k}\gamma_{s,k}}{1+P_{s,k}\gamma_{s,k}}\Big]\Big\}\\ &+(1+\mu^*)\mathbb{E}_{\gamma^j_{k,d}|\hat{\gamma}^j_{k,d}}\Big[\frac{P_{k,d}\gamma_{k,d}}{1+\sum_k^K P_{k,d}\gamma_{k,d}}\Big]\Big),\end{aligned} \tag{18}$$

Since we know that $\hat{\gamma}^i_{s,k}=\frac{L_{s,k}\hat{G}^i_{s,k}}{\sigma_k^2}$. The channel SNR $\gamma_{s,k}$ conditioned on $\hat{\gamma}^i_{s,k}$ is also a non-central Chi-squared distributed random variable with PDF:

$$f(\gamma^i_{s,k}|\hat{\gamma}^i_{s,k})=\frac{1}{\nu^i_{s,k}}e^{-\frac{\hat{\gamma}^i_{s,k}+\gamma^i_{s,k}}{\nu^i_{s,k}}}\mathcal{J}_0\Big(2\sqrt{\frac{\hat{\gamma}^i_{s,k}\gamma^i_{s,k}}{(\nu^i_{s,k})^2}}\Big) \tag{19}$$

$$f(\gamma^i_{s,k}|\hat{\gamma}^j_{k,d}) = \frac{1}{\nu^j_{k,d}} e^{-\frac{\hat{\gamma}^j_{k,d}+\gamma^j_{k,d}}{\nu^j_{k,d}}} \mathcal{J}_0\left(2\sqrt{\frac{\hat{\gamma}^j_{k,d}\gamma^j_{k,d}}{(\nu^j_{k,d})^2}}\right) \tag{20}$$

where $\nu^i_{s,k} = \sigma_k^2/\sigma^2_{\tilde{h}}$ and $\nu^j_{k,d} = \sigma_d^2/\sigma^2_{\tilde{h}}$. Following same procedure as power allocation, we obtain

$$\begin{aligned}\mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}}\left[\frac{P^i_{s,k}\gamma^i_{s,k}}{1+P^i_{s,k}\gamma^i_{s,k}}\right] =& \psi^i_{s,k}\big(\frac{\theta^i_{s,k}}{P^i_{s,k}}\big)^{\psi^i_{s,k}} e^{\frac{\theta^i_{s,k}}{P^i_{s,k}}} \\ & \Gamma\big(-\psi^i_{s,k}, \frac{\sigma_k^2\theta^i_{s,k}}{P^i_{s,k}}\big),\end{aligned} \tag{21}$$

$$\begin{aligned}\mathbb{E}_{\gamma^j_{k,d}|\hat{\gamma}^j_{k,d}}\left[\frac{P^j_{k,d}\gamma^j_{k,d}}{1+\sum_k^K P^j_{k,d}\gamma^j_{k,d}}\right] =& \psi^j_{k,d}\big(c_2\theta^j_{k,d}\big)^{\psi^j_{k,d}} e^{c_2\theta^j_{k,d}} \\ & \Gamma\big(-\psi^j_{k,d}, c_2\theta^j_{k,d}\big),\end{aligned} \tag{22}$$

where $\psi^i_{s,k} = (\zeta^i_{s,k}+1)^2/(2\zeta^i_{s,k}+1)$ with $\zeta^i_{s,k} = \hat{\gamma}^i_{s,k}/\nu^i_{s,k}$ and $\theta^i_{s,k} = \zeta^i_{s,k}/(\hat{\gamma}^i_{s,k}+\nu^i_{s,k})$. $\psi^j_{k,d} = (\zeta^j_{k,d}+1)^2/(2\zeta^j_{k,d}+1)$ with $\zeta^j_{k,d} = \hat{\gamma}^j_{k,d}/\nu^j_{k,d}$ and $\theta^j_{k,d} = \zeta^j_{k,d}/(\hat{\gamma}^j_{k,d}+\nu^j_{k,d})$. We have $c_2 = (1+\sum_{m=1,m\neq k}^{K} P_{m,d}\gamma_{m,d})/P^j_{k,d}$. $P_{m,d}$ and $\gamma_{m,d}$ are the power allocation and channel SNR from relay m to MT. Optimal value of $\mathbf{P}$ can be given in (15) and (16). Thus, (18) can be viewed as multi-objective optimization problem, which aims at obtaining the trade-off of the throughputs of first hop and second hop. (18) is also the termination criteria for the whole RRA scheme. Therefore, the relay selection strategy is

$$\rho_k = \begin{cases} 1 & \text{if } k \in \mathcal{K}^*, \\ 0 & \text{otherwise.} \end{cases}$$

D. Optimal Subcarrier Allocation (OSA)

The goal of subcarrier allocation strategy is to assign subcarriers to a given RN that can obtain best throughput performance. Following the same procedure as the relay selection, we could obtain subcarrier allocation criteria as follows:

$$\mathcal{I}^* = \arg\max \left\{\min_{k\in\mathcal{K}}\left\{(1-\mu^*)\mathbb{E}_{\gamma^i_{s,k}|\hat{\gamma}^i_{s,k}}\left[\frac{P^i_{s,k}\gamma^i_{s,k}}{1+P^i_{s,k}\gamma^i_{s,k}}\right]\right\}\right. \tag{23}$$

$$\mathcal{J}^* = \arg\max\left\{(1+\mu^*)\mathbb{E}_{\gamma^j_{k,d}|\hat{\gamma}^j_{k,d}}\left[\frac{P^j_{k,d}\gamma^j_{k,d}}{1+\sum_k^K P^j_{k,d}\gamma^j_{k,d}}\right]\right\}, \tag{24}$$

where channel SNR $\gamma^i_{s,k} = \frac{L_{s,k}G^i_{s,k}}{\sigma_k^2}$ and $\gamma^j_{k,d} = \frac{L_{k,d}G^j_{k,d}}{\sigma_d^2}$. Therefore, the OSA indicator for the first hop and second hop can be expressed as

$$\omega_i = \begin{cases} 1 & \text{if } i \in \mathcal{I}^*, \\ 0 & \text{otherwise.} \end{cases}$$

$$\omega_j = \begin{cases} 1 & \text{if } j \in \mathcal{J}^*\ , \\ 0 & \text{otherwise.} \end{cases}$$

IV. PERFORMANCE EVALUATION

Simulations are presented to evaluate the performance of proposed algorithms in this section. It is assumed that five RNs are located between AP and MT, and MT is 1.8km away from AP. The Stanford University SUI-3 channel model is employed [16], in which the central frequency is 1.9GHz. Channel is assumed to be 3-tap channel and signal fading follows Rician distribution. We choose number of subcarriers N to be 32, so the duality gap can be ignored [6]. Flat quasi-static fading channels are considered, hence the channel coefficients are assumed to be constant during a complete frame, and can vary from a frame to another independently. The noise variance of the two hops are set to be 1 for simplicity. The path loss factor varies according to the different distances from RNs to AP and MT. If distance between RN and AP or RN and MT is shorter than a break point $d_{BP} = 100m$, the exponent is fixed to 2, otherwise it is 3.5. The maximum transmit power of AP and RN are 40 dBm and 20 dBm respectively. An accurate MMSE estimator with estimated variance $\sigma^2_{\tilde{h}} = 0.02$ is assumed at the receiver.

We demonstrate our results comparing with the performance of some other schemes:

1) Equal power allocation combined with proposed subcarrier allocation scheme and relay selection(EPA);

2) Waterfilling power allocation and proposed subcarrier allocation scheme with relay selection (Waterfilling);

3) Modified proportional allocation scheme in [5] with fairness consideration(Fairness SA);

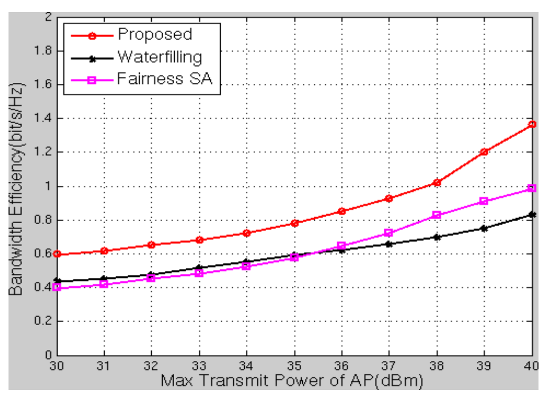

Figure 3. Impact of maximum transmit power $P_{s,max}$ on system bandwidth Efficiency

Fig. 3 demonstrates the impact of maximum transmit power of AP on the system bandwidth efficiency. We denote $D_{s,d}$ as the distance between AP and MT, and $D_{s,k}$ as the distance between AP and RNs. In Fig. 3, we have $D_{s,d} = 1800$ m and $D_{s,k}$ from 1500 m to 1600 m. The considered channel SNR at the RN k is varied from $\gamma_{s,k} = -20dB$ to $\gamma_{s,k} = -30dB$ and at MT d it is varied from $\gamma_{k,d} = -15dB$ to $\gamma_{k,d} = -25dB$. It can be seen that the proposed scheme achieves the best performance. The performance gain over other methods in comparison is up to 20%. It can also be noticed that if Waterfilling is used as the power allocation scheme (instead of our proposed scheme), the throughput performance is comparable with Fairness SA. Another performance gain can be seen in power consumption. We can see that with a fixed data rate requirement, our proposed scheme provides a clear power saving gain. For instance, at the level of 1.0 bit/s/Hz bandwidth efficiency our proposed scheme can reach a power saving around 3 dBm compared to the other schemes.

Fig. 4 shows the impact of distance between AP and RN on the system throughput. The distance between AP and RN is normalized to distance between AP and MT and vary from 0.1 to 0.9. In Fig. 4, we set maximum AP power $P_{s,max} = 40\ dBm$ and maximum RN power is $P_{k,max} = 20\ dBm$. From Fig. 4, we can see that the proposed algorithm obtain the highest system capacity when distance is less than 0.9 in Fig. 4. When the average normalized distance between AP and RN is around 0.9, we can find that performance difference between EPA and proposed scheme are smaller. This may due to the fact the some RNs are very close to the MT so that the achieved SNR is rather high. It can be concluded that the proposed algorithm can provide better performance gain over other existed algorithms.

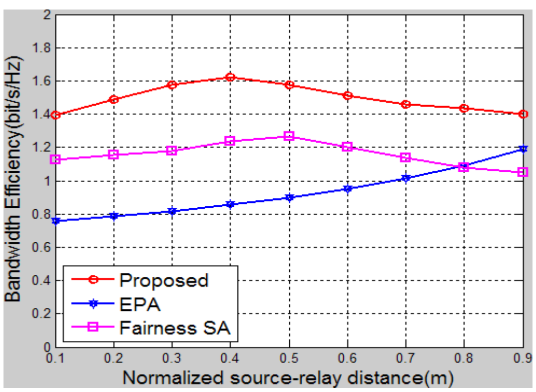

Figure 4. Impact of distance between AP and RN on the system bandwidth efficiency, maximum AP power is 40 dBm, maximum RN power is 20 dBm

Fig. 5 illustrates the convergence speed of the proposed algorithm and the Fairness SA scheme. The considered $P_{s,max}$ is fixed to 35 dBm and $P_{r,max}$ is 17.5 dBm. Our proposed algorithm reaches the steady state after several iterations, which demonstrates fast convergence speed.

V. Conclusion

In this paper we investigated the problem of resource allocation for cooperative multi-relay assisted OFDMA networks with assumption of Imperfect CSI. The joint optimization problem for radio resource allocation was solved by addressing three subproblems including optimal selection of collaborative relays, subcarriers and power with the objective of maximizing the expected system throughput. Theoretical expressions were derived for the optimal selections. It was shown that by designing RRA scheme with imperfect CSI for different hops, it is possible to reach a noticeable gain in the cell-edge throughput. In addition, the results support our theoretical analysis that the proposed scheme obtain power allocation as close to the one when perfect knowledge of CSI is considered. These were also illustrated with simulation examples.

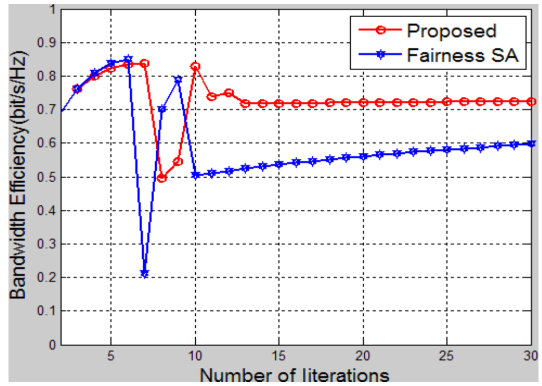

Figure 5. Impact of the number of iteration on the system bandwidth efficiency

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