# Service Provisioning and User Association for Heterogeneous Wireless Railway Networks 

Yun Hu, Student Member, IEEE, Zheng Chang, Member, IEEE, Hongyan Li, Member, IEEE, Tapani Ristaniemi, Senior Member, IEEE, and Zhu Han, Fellow, IEEE


#### Abstract

In addition to comforting passengers' journey, modern railway system is responsible to support a variety of on-board Internet services to meet the passenger's demands on seamless service provisioning. In order to provide wireless access to the train, one idea attracting increasing attention is to deploy a series of track-side access points (TAPs) with high-speed data rates along the rail lines dedicated to the broadband mobile service provisioning on board. Due to the heavy data traffic flushing into the base stations (BSs) of the cellular networks, TAPs act as a complement to the BSs in data delivery. In this paper, we focus on the TAP association problem for service provisioning in a heterogeneous wireless railway network where the TAP and BS coexist by applying a queueing game theoretic approach. Specifically, we present comprehensive theoretical analysis of the delay performance on the circumstances of partially observed, totally unobserved and totally observed the state of the system. Moreover, based on the considered payoff model and the derived association delay time, the passenger's equilibrium strategies on association behaviors, i.e. whether to associate with a TAP or not, are studied. Finally, performance evaluations and discussions are provided to illustrate our proposed passenger-TAP association scheme for the heterogeneous wireless railway communication system.


Index Terms-Service provisioning, game theory, delay time, railway networks, heterogeneous wireless network.

## I. Introduction

WIRELESS railway system has been developed rapidly all over the world in the past few years. It is not only dedicated to providing the passengers fast, convenient and comfortable travel experience, but also satisfying their ever-increasing demand for a variety of broadband mobile communication on board [1]. By constructing a heterogeneous network architecture [2], a series of track-side access points (TAPs) [3], [4] complement the base stations (BSs) of the cellular networks in order to provide high-speed data rates for

[^0]the on-board Internet services. However, due to the limited transmission power, the TAPs can only support intermittent wireless communication coverage for the rail line. What is more, the high-speed moving train brings about heavy Doppler effects to the wireless communication [5], [6]. Under the circumstances, the data transmission rates of the wireless channel will change dynamically in real time. Therefore, it is extremely crucial to study on the communication technology for the railway communication system (RCS) [7].

According to the characteristics of TAPs, how to satisfy various data services of railway communication is a hot issue in recent years. Since TAPs only provide limited data transmission rates and queueing buffers, the delay time that a passenger experiences is considered as one of the most critical issues in the RCS. In [8], a tight delay bound of train control services is proposed by applying the stochastic network calculus for the queueing system. To satisfy the passenger's demand, the delay performance of the Internet multimedia services also deserves much attention in the heterogeneous wireless railway network. By analyzing the wireless communication on BSs via access points, an optimal power allocation strategy is investigated in [9] on account of delay requirements. Nevertheless, this work is constraint to uplink transmission in the high-speed railway scenario. In [10], a delay analysis model is proposed by utilizing queueing theory in a discrete time form as well as a delay propagation model in high-speed train communication scenario. But it only considers the delay performance through switch ports in carriages. Given delay requirements, [11] formulates a resource allocation problem into a stochastic optimization problem in high-speed railway wireless communication. In recent literatures, the delay performance of the ondemand data delivery is seldom analyzed in the RCS, which may bring inaccurate results. For example, some solutions extracted from vehicular environments [12], such as the data packets delivery between the vehicles and the highway, cannot be directly applied to the railway communication. [13] models a radio resource scheduling mechanism as an infinite-horizon average cost Partially Observed Markov Decision Process (POMDP) in order to optimize the delay performance through stochastic learning in vehicular networks. It does not contain the situation that the system information is totally unobserved, which is a non-trivial problem. In our previous papers [14], [15], we analyze the end-to-end delay bounds of two kinds of data applications through one server under the heterogeneous network for high-speed trains. However, the cooperation of the TAPs and BSs is not taken into account in both works. Therefore, it is urgent to focus on the impact of the queueing delay performance in the RCS.

During the past few decades, queueing theory [16] has been researched from an economic view [17] with more and more interests. The passengers can make their choices on whether to join the queue or not, which represents the passenger is willing to wait in the queue for service or to abandon to be served in order to maximize their own benefits. In this way, it is easy to construct a reward-cost framework in order to reflect the passenger's decision on whether he is willing to wait in the system or not. Nevertheless, there exists such a situation that some passengers possess the same objective for data services in the RCS [18]. On this occasion, it is necessary to research on the queueing theory by utilizing the game theory to obtain the equilibrium behaviors for passengers on the train. In [19], the state-of-the art of game-theoretic analysis is presented based on the pricing strategy as well as the impact on the passenger's equilibrium strategy with an economic view for the queueing system. In a monopolist system, [20] proposes the uniform pricing and the priority auction mechanisms with consideration of the delay sensitive customers. The customer can make his/her own decision on whether to join or balk the queue according to the server's pricing mechanism in a Stackelberg game. The sojourn time is the key measurement for the delay cost of a customer for the service. [21] researches on the equilibrium balking strategies obtained under an observable queue case with server vacations due to random breakdowns and repairs. The vacation length is assumed to be independent of the queue size. The customers can make his/her own decision by observing the queue size. However, it does not consider the impact of the queueing delay on the single-server queue.

As far as we know, most of the previous works apply the queueing game theory to wireless networks, especially to cognitive radio networks [22]. [23] analyzes a queueing system where one customer transmitting a data packet will interference all other customers since they share a range of the wireless spectrum. A cross-layer algorithm is proposed to solve the problem that whether to transmit or to stay in the queue in order to maximize the network throughout. However, it does not consider the queueing delay effects in the system to make the decision by each customer. [24] investigates a dynamic on-off strategy based on a tradeoff by considering the secondary delay and the power consumption. The socalled secondary delay is caused by the primary data packets relaying through the secondary node. [25] studies the queueing control with random service interruptions in cognitive radio networks. By equalizing the individually and socially optimal strategies, whether a data packet should join the queue or not can be decided by an optimal threshold of the queue length. However, this kind of queueing control model cannot be used to analyze various traffic metrics. What is more, its channel statistics are unknown to the system, which is not suitable for the heterogeneous wireless railway network. [26] concentrates its attention on the coupling of pricing, load balancing and the secondary user's spectrum access decision. Jointly optimal pricing and load balancing problem can be solved by optimally characterizing the customer's economic issue and their spectrum access decision. A secondary user can make its own decision strategically based on its perceived queueing
delay. An unobservable queueing system is constructed in [27] to propose the socially optimal pricing schemes. The service provision to customers are decided by the pricingbased methods.
In our previous paper [28], it has attracted our interest as how to allocate the spectrum access for the secondary user. We investigate a queueing game with pricing strategies that the secondary user can make his own decision on whether it should join the partially observed queue of the BS or not based on the queue information and its own payoff. In order to optimize the social welfare of the whole system, the individual equilibrium strategy keeps consistent with the social optimal strategy. However, the social optimal strategy is not suitable for the heterogeneous wireless railway networks. Moreover, the data transmission rates of the TAP are much higher than the BS. It is impractical and inefficient to access to the BSs for the railway networks. Unlike some existing literatures which only consider the queueing delay, the experienced delay by the passengers includes the queueing delay plus the serving delay in this paper.

Motivated by the observed problems and inspired by the previous works, the contribution of this work can be summarized as follows:

- In the system model, we study the case that there are several TAPs widely deployed along a pre-defined rail line. The deployment of the TAPs can be considered as a complementary part in addition to the cellular networks for providing the passengers wireless connection to the Internet. Specifically, by considering the characteristics of the high data transmission rates and intermittent connections for the TAP wireless link, it is assumed that when the train is moving along the rail line, the passengers can choose to be connected with the TAPs intermittently to request on-board services.
- A symmetric game among the passengers is proposed to maximize their own payoffs. The formulation of the individual payoff model for each passenger consists of three elements, i.e., players, reward and cost. The reward for the passenger is related to the achievable data rate and the cost is affected by the delay time they may encounter in the queue of the TAP. We present a payoff function to facilitate the passenger to decide whether to associate with the TAP or not during the data transmission.
- By applying the queueing and game theoretic-based approaches, our goal is to investigate the equilibrium strategies of the passengers in the railway system. Based on some previous analysis and results in [19] and [20], we consider four different practical scenarios according to the visibility of information level of the TAP's queue, i.e., the queue length and the train position status, and examine their impacts on the association decisions of the passengers. Moreover, we present a fundamental analysis on the delay performance and accordingly derive the passenger's equilibrium strategy under four cases. Performance evaluation and discussions are also presented to illustrate the proposed scheme.
The reminder of this paper is organized as follows. The


Fig. 1. System Model
system model and assumptions are presented in Section II. In Section III, queueing analysis and equilibrium strategy for user association is given. Performance evaluations and discussions are presented in Section IV. At last, we conclude this work in Section V.

## II. System Model

In this section, we present our system model together with some necessary assumptions, such as the network configuration, train movement trajectory, as well as data arrival process and service process.

## A. Network Configuration

The system model is presented in Fig. I. In the considered system, the TAPs are deployed along the rail line and they can only be accessed to the train within intermittent time periods. Thus, the passengers on the train are able to connect with the backbone internet via TAP when the train is in the transmission range of the TAPs. In order to concentrate on the delay analysis and equilibrium strategy on the wireless connection, it is assumed that the packet delay on the wired line from the backbone to the TAPs can be neglected. In the system, we assume that a central controller is responsible for allocating network resources based on the data traffic demands and the train trajectory. Thus, when the train is in the transmission range of the TAPs, the on-demand service request can be proceeded by the TAPs. In this context, the requests of the passengers can enter the queue of the TAP and wait to be served.

## B. Train Movement Trajectory

In general, for a railway system, the location and the speed of the train can be obtained in advance. Therefore, the ondemand services can be delivered with high accuracy from the Internet server to the passengers on the train. The accuracy of the train movement trajectory is important for analyzing the service provisioning. For example, in Fig. I, within a time duration $\left[T_{s}, T_{e}\right]$, the train can travel from the origin station to the destination station. During the trip, TAPs are deployed along the rail line. For simplicity, in Fig. I, we take three TAPs as an example. When the three TAPs are deployed, there
exist three separated contact durations for the passengers to deliver data packets, which can be represented as $\left[T_{s}^{i}, T_{e}^{i}\right]$, $i \in[1, \ldots, I]$. Here, $T_{s}^{i}$ and $T_{e}^{i}$ denote the starting time and the ending time of data delivery within the $i$ th TAP, respectively. We assume that $T_{s}^{i} \leq T_{e}^{i}, T_{s} \leq T_{s}^{1}, T_{e} \geq T_{e}^{I}$ for $i \in[1, \ldots, I]$.

## C. Data Arrival Process and Service Process

Considering the above defined network configuration and train movement trajectory model, the passengers with service requests are to make the decision on whether to associate with the TAP or not. When the data of the passenger cannot be queued in the buffer of the TAP, we assume that it can be delivered to the cellular BS if needed ${ }^{1}$. In practice, as one passenger may have multiple packets for transmission, we use customer instead of passenger/packet in the following queue analysis. The customer can be considered as data packets, sessions or connections in this queueing model [25]-[28]. Different customers could be from different passengers or from the same passenger. Note that when considering decision making process, we use customer and passenger interchangeably.

The customer arrival rate at the TAPs is assumed to follow a Poisson process at rate $\lambda$ [29] and it is assumed to be independent and identical distribution (i.i.d.) across time slots. To make the analysis simple, by referring to the vehicular networks in [30], we model a M/M/1 queueing system at the TAP where its service rate is assumed to be i.i.d. with exponential distribution $\mu$. The analysis can be also generalized to a M/G/1 queueing model in which the service time is assumed to follow a general independent distribution. In this paper, we just provide an intuitive perspective to the association problem in the heterogeneous wireless railway networks. To satisfy the customers' demand, most of the TAPs should be deployed along the rail line with heavy data traffic and few TAPs are deployed in the rural areas to save the deployment cost. Without loss of generality, while considering the obstacle of practical wireless environment, we assume that $T_{s}^{i+1}-T_{e}^{i}$ which is the time a train travels between two isolated TAPs is exponentially distributed at rate $\xi$ and the serving time of a TAP $T_{e}^{i}-T_{s}^{i}$ is according to a exponential process at rate $\theta$. The queue is considered to be stable so we have $\mu \theta>\lambda(\theta+\xi)$. The service order of the data is assumed to follow the First-Come-First-Served (FCFS) rule. In addition, when the train is out of the transmission range of a TAP, the information of the queue will be transferred to the next TAP.

We denote the state of the queue at TAP at a time slot $t$ is as a pair $(N(t), I(t))$, which consists of the length of queue $N(t)$, i.e., the number of customers in the system, and the train position status $I(t)$. If the train is in the coverage of the TAP, $I(t)=1$ and otherwise, $I(t)=0$.

## III. Queueing Analysis and Equilibrium Strategy FOR USER AsSOCIATION

By comparing with the BS wireless link, the TAP wireless link has much higher data transmission rates in the RCS.

[^1]Therefore, the TAP consumes less delay for the customers in data delivery. In this way, we consider that the TAP can offer better QoS than the BS. To investigate the association and service provisioning solution, we assume that the customer can receive a reward after being successfully served by the TAP. Meanwhile, we also assume that while waiting in the queue, the cost of a customer is a function of its waiting time. Correspondingly, the customer will make an irreversible decision on whether to associate with the TAP or not based on the expected reward and cost, i.e., the customer cannot quit until being served after associating with the TAP. Since the TAPs can only provide limited communication range due to its constraint transmission power, there are huge gaps between adjacent TAPs. The arrived requests from the customers will form a queue and then decide whether they should be served by the TAP. In this way, the queued packets have to endure remodeling process from time to time during the train movement. It is reasonable to construct a queue with vacations in the high speed train scenario. We refer to the difference between the reward and cost as the payoff of the customers.

In this section, the goal is to investigate that under what circumstance, the customer can make a decision on associating with the TAP and wait in the queue of the TAP. The customers are considered to be risk neutral ${ }^{2}$. Therefore, the customers aim to maximize their own payoffs and a symmetric game among the customers is considered in this work. Intuitively, in the considered game model, a strategy is an equilibrium if it is the best response against itself. Moreover, we also take into consideration of whether the customers are aware of the information of the queue, i.e., a customer may not be aware of some of the system parameters upon arrival which is rather practical for a wireless communication system. At first, we investigate the perfect case that both $N(t)$ and $I(t)$ are known to the incoming customer at time $t$. Then, we also consider the case that the length of the queue $N(t)$ is known to the customer upon arrival, and the state of the TAP $I(t)$ remains unknown. In addition, we further explore the reverse situation that $I(t)$ is known at time slot $t$ but $N(t)$. Last but not the least, we also investigate the case that $(N(t), I(t))$ are both unknown to the arriving customers due to the system limitation.

## A. Payoff Model

We first present our definitions of the players, rewards and cost in the considered game model.

- Player: The player of the symmetric game denotes the customer in the train who has data packets to be delivered.
- Reward: The customer can obtain a reward $\varphi_{s}$ after being served by the TAP. The reward can be considered as the satisfaction level or any other benefits obtained from the service. For example, a utility function associated with the achievable data rate can be considered.

[^2]- Cost: The cost of the customer is represented by $\chi(T)$ where $T$ is waiting time plus the serving time in the queue of the TAP, i.e., the experienced delay. Generally, $\chi(T)$ should be considered as an increasing function of $T$. In this work, we advocate a linear example to simplify the analysis. By considering $C$ as the unit cost, we assume that $\chi(T)=C T$. We assume that $R$ is positive for $n=0$ in order to avoid the trivial case when $n=0$ and $R=0$, which means

$$
\begin{equation*}
\varphi_{s}>\frac{C}{\mu}\left(1+\frac{\xi}{\theta}\right)+\frac{C \xi}{\theta(\lambda+\theta+\xi)} \tag{1}
\end{equation*}
$$

Then we can utilize a generic payoff model of the customer in Definition 1, which is commonly refereed in the queueing analysis [19], [27].

Definition 1. The payoff of a single customer is modelled as [19]

$$
\begin{equation*}
R:=\varphi_{s}-C T \tag{2}
\end{equation*}
$$

With the movement of the train, the queue information of one TAP can be forwarded to the next TAP. And accordingly, the theoretical result of $T$ is derived. Therefore, the payoff function (2) depends on the associated TAP and is defined as a function of the service and waiting time $T$, which is related to the queue status and the association decision of the customers.

## B. Queueing Analysis and Equilibrium Strategy Analysis

1) Case 1: Both $I(t)$ and $N(t)$ are known
a) Queueing Analysis

In this case, we utilize the results in [21] and consider the case both $N(t)$ and $I(t)$ are observable to the customers with data requests. Since whether the train in the coverage of a TAP and the number of the customers waiting in the queue are known to the incoming customer, a pure threshold strategy (PT1), instead of a mixed strategy, is specified by a pair $\left(n_{e}(0), n_{e}(1)\right) \cdot n_{e}(I(t))$ is the threshold of the queueing length for the customer with data requirement to decide whether to associate or not. The PT1 can be defined as "While arriving at time $t$, observe $(N(t), I(t))$; Associate with the TAP if $N(t) \leq n_{e}(I(t))$; Otherwise, remain in the cellular networks." Therefore, the expected waiting time of a customer can be given in Proposition 1.

Proposition 1. A customer finds the expected waiting time just before its arrival given as

$$
\begin{equation*}
T(n, i)=(n+1)\left(1+\frac{\xi}{\theta}\right) \frac{1}{\mu}+(1-i) \frac{1}{\xi} . \tag{3}
\end{equation*}
$$

Proof: The proof can be found in Appendix A.
b) Equilibrium Strategy Analysis

According to Proposition 1, the threshold $\left(n_{e}(0), n_{e}(1)\right)$ can be presented in Theorem 1.

Theorem 1. Under the assumption of (1), in the queue of the TAP, PT1 is a weakly dominant strategy for existed a pair of

## thresholds

$$
\begin{equation*}
\left(n_{e}(0), n_{e}(1)\right)=\left(\left\lfloor\frac{\varphi_{s} \mu \theta-C \mu}{C(\xi+\theta)}\right\rfloor-1,\left\lfloor\frac{\varphi_{s} \mu \theta}{C(\xi+\theta)}\right\rfloor-1\right) \tag{4}
\end{equation*}
$$

Proof: Based on the expected waiting time in (3), a customer who enters the queue with state $(n, i)$ has a payoff as follow,

$$
\begin{equation*}
R(n, i)=\varphi_{s}-C T(n, i) \tag{5}
\end{equation*}
$$

From (5), we can see that if $R(n, i)>0$, a customer will associate with the TAP. Since the TAP acts as a complementary role in the cellular networks, it is dedicated for providing different kinds of on-board services for the customers in the heterogeneous wireless railway networks. In this paper, we assume that the customers are connected to the BSs in a natural way. So, if it is beneficial to connect to the TAP, the customers choose to associate with the TAP. We consider the payoff value is positive in this case. However, if the customers decide not to associate with the TAP, they will seek for or stay at the BS. In this case, the payoff for associating with the BS is assumed to be less than 0 . If $R(n, i)=0$, the customer is indifferent between associating and balking. Therefore, by addressing $R(n, i)=0$ for $n$, we can arrive that the customer with requests will associate with the TAP if and only if the queue length $n=n_{e}(i), \forall i \in\{0,1\}$, and $\left(n_{e}(0), n_{e}(1)\right)$ can be found in (4). This strategy is preferable, independently of what the other customers do, i.e., it is a weakly dominant strategy.
2) Case 2. $I(t)$ is unknown and $N(t)$ is known
a) Queueing Analysis

To this end, we then explore the equilibrium strategy when only $I(t)$ is not informed to the customer. Meanwhile, $N(t)$ is known to the arriving customer ${ }^{3}$. Accordingly, as the customer is able to obtain the information of $N(t)$, the pure threshold strategy (PT2) about a threshold $n_{e}$ exists and we define it as follows:

While the customer arrives at time $t$, it will be informed about $N(t)$ and $I(t)$ is unknown. If $N(t) \leq n_{e}$, the customer will associate with the TAP. Otherwise, if $N(t)>n_{e}$, the customer remains in the cellular networks.

Accordingly, first we can obtain the expected mean delay time of an arriving customer with data requests in Proposition 2.

Proposition 2. When a customer associates with the TAP according to a threshold strategy PT2, the mean delay of a customer that observes $n$ customers ahead and decides to associate with the TAP can be given in (6) and (7),

$$
\begin{align*}
& E_{1}(n)=\frac{n+1}{\mu}\left(1+\frac{\xi}{\theta}\right)+\frac{\left(x_{1} / x_{2}\right)^{(n+1)}-1}{\theta\left(\left(1+\beta_{1}\right)\left(x_{1} / x_{2}\right)^{(n+1)}-\left(1+\beta_{2}\right)\right)}  \tag{6}\\
& n \in\left\{0,1, \ldots, n_{e}\right\}
\end{align*}
$$

[^3]\[

$$
\begin{align*}
& E_{2}(n+1)=\frac{n+2}{\mu}\left(1+\frac{\xi}{\theta}\right) \\
& +\frac{\left(\mu+\xi\left(1+\beta_{1}\right)\right)\left(x_{1} / x_{2}\right)^{(n+1)}-\left(\mu+\xi\left(1+\beta_{2}\right)\right)}{\theta\left(\left(\mu+(\xi+\theta)\left(1+\beta_{1}\right)\right)\left(x_{1} / x_{2}\right)^{(n+1)}-\left(\mu+(\xi+\theta)\left(1+\beta_{2}\right)\right)\right)} \tag{7}
\end{align*}
$$
\]

where $x_{1}, x_{2}$ and $\beta_{j}$ are given as

$$
\begin{gather*}
x_{1}=\frac{\lambda}{2 \mu(\lambda+\theta)}(\mu+\xi+\lambda+\theta+ \\
\left.\sqrt{(\mu+\xi+\lambda+\theta)^{2}-4 \mu(\lambda+\theta)}\right) \\
x_{2}=\frac{\lambda}{2 \mu(\lambda+\theta)}(\mu+\xi+\lambda+\theta-  \tag{8}\\
\left.\sqrt{(\mu+\xi+\lambda+\theta)^{2}-4 \mu(\lambda+\theta)}\right) \\
\quad \beta_{j}=\frac{(\lambda+\theta) x_{j}-\lambda}{\xi x_{j}}, j \in\{1,2\} .
\end{gather*}
$$

Proof: The proof is given in Appendix B.

## b) Equilibrium Strategy Analysis

Next, we need to analyze the stationary distribution when the customers with data requests follow PT2 in Proposition 3.
Proposition 3. The stationary probability $p(n, i), n \in$ $\left\{0,1, \ldots, n_{e}+1\right\}, i \in\{0,1\}$ can be given as follows,

$$
\begin{align*}
& p(n, 0)=\alpha\left(x_{1}^{n+1}-x_{2}^{n+1}\right), n \in\left\{0,1, \ldots, n_{e}\right\} \\
& p(n, 1)=\alpha\left(\beta_{1} x_{1}^{n+1}-\beta_{2} x_{2}^{n+1}\right), n \in\left\{0,1, \ldots, n_{e}\right\} \\
& p\left(n_{e}+1,0\right)=  \tag{9}\\
& \quad \frac{\alpha \lambda}{\theta}\left(1+\frac{\xi}{\mu}\left(1+\beta_{1}\right)\right) x_{1}^{n_{e}+1} \\
& \\
& \quad-\left(1+\frac{\xi}{\mu}\left(1+\beta_{2}\right)\right) x_{2}^{n_{e}+1} \\
& p\left(n_{e}+1,1\right)=
\end{align*}
$$

Proof: In order to obtain the stationary distribution, the following balance equations can be used [21],

$$
\begin{gather*}
(\lambda+\theta) p(0,0)=\xi p(0,1)  \tag{10}\\
(\lambda+\theta) p(n, 0)=\xi p(n-1,0)+\xi p(n, 1), n \in\left\{0,1, \ldots, n_{e}\right\}  \tag{11}\\
\mu p(n+1,1)=\lambda p(n, 0)+\lambda p(n, 1), n \in\left\{0,1, \ldots, n_{e}\right\}  \tag{12}\\
\theta p\left(n_{e}+1,0\right)=\lambda p\left(n_{e}, 0\right)+\xi p\left(n_{e}+1,1\right) \tag{13}
\end{gather*}
$$

Addressing (11) with respect to $p(n, 1)$ and substituting in (12), one can obtain

$$
\begin{align*}
& \mu(\lambda+\theta) p(n+1,0)-\lambda(\lambda+\mu+\theta+\xi) p(n, 0)  \tag{14}\\
& +\lambda^{2} p(n-1,0)=0, n \in\left\{0,1, \ldots, n_{e}-1\right\}
\end{align*}
$$

(14) is a homogeneous second-order difference equation and the solution is

$$
\begin{equation*}
p(n, 0)=c_{1} x_{1}^{n}+c_{2} x_{2}^{n}, n \in\left\{0,1, \ldots, n_{e}\right\} \tag{15}
\end{equation*}
$$

where $x_{1}$ and $x_{2}$ are the roots of the corresponding characteristic equation. The expression of $x_{1}$ and $x_{2}$ can be found in (8). $c_{1}$ and $c_{2}$ are constants to be determined. Substituting (15) into (11), we can obtain

$$
\begin{equation*}
p(n, 1)=c_{1} \beta_{1} x_{1}^{n}+c_{2} \beta_{2} x_{2}^{n} \tag{16}
\end{equation*}
$$

where $\beta_{i}, i=1,2$ is as presented in (8). By substituting (15) and (16) into (10) and (12), respectively, one can obtain $c_{2} / c_{1}=-\beta_{2} / \beta_{1}$. Then, the unique unknown constants $c_{i}, \forall i \in\{1,2\}$ can be derived using the normalization equation
as an explicit but involved sum. We can express $\alpha$ as

$$
\begin{equation*}
\alpha=\frac{c_{1}}{\beta_{1}}, \tag{17}
\end{equation*}
$$

and Proposition 3 can be proved.
Based on the obtained expected mean delay time, the payoff of the customer with data requests can be derived. Now the pure threshold equilibrium strategies can be derived with the above two propositions and Theorem 2 is explained as follow.

Theorem 2. Under the assumption of (1), PT2 is a symmetric Nash equilibrium strategy for $n_{e} \in\left\{n_{l}, n_{l}+1, \ldots, n_{u}\right\}$, where $n_{l}$ is the lower bound and $n_{u}$ represents the upper bound of $n_{e}$. The properties of $n_{l}$ and $n_{u}$ are presented in (19) and (20).

Proof: First, two sequences, $g_{1}(n)$ and $g_{2}(n)$ are defined by

$$
\begin{align*}
& g_{1}(n)=\varphi_{s}-C E_{1}(n), n \in \mathbb{N}^{0} \\
& g_{2}(n)=\varphi_{s}-C E_{2}(n), n \in \mathbb{N}^{0} \tag{18}
\end{align*}
$$

In addition, we also consider $n_{l}, n_{u} \in \mathbb{N}^{0}$, where $n_{l} \leq$ $n_{u}$. Because of the assumption of $R, g_{1}(0)>0$ and $\lim _{n \rightarrow+\infty} g_{1}(n)=-\infty$, thus, if we use $n_{u}+1$ as the subscript of the first non-positive of $g_{1}(n)$, one can arrive

$$
\begin{equation*}
g_{1}(0), g_{1}(1), g_{1}(2) \ldots, g_{1}\left(n_{u}\right)>0, g_{1}\left(n_{u}+1\right) \leq 0 \tag{19}
\end{equation*}
$$

Moreover, we also find that $g_{1}(n)>g_{2}(n), \forall n \in \mathbb{N}^{0}$. Correspondingly, one can obtain $g_{2}\left(n_{u}+1\right)<g_{1}\left(n_{u}+1\right) \leq 0$. If $n_{l}$ is assumed to be the first subscript that $g_{2}(n) \geq 0$, we have

$$
\begin{equation*}
g_{2}\left(n_{l}+1\right), g_{2}\left(n_{l}+2\right), \ldots, g_{2}\left(n_{u}+1\right)<0, g_{2}\left(n_{l}\right) \geq 0 \tag{20}
\end{equation*}
$$

We have the model where the customers follow PT2 for fixed value of $n_{e} \in\left\{n_{l}, n_{l}+1, \ldots, n_{u}\right\}$. By using (6) and (7), we can obtain the payoff of the customer, if it arrives at time $t$ and finds there are $n$ customers ahead and decides to send the request to the queue. According to the payoff and the definitions in (18) and (19), the customer will associate with the TAP when there are $n \leq n_{e}$ waiting customers in the queue and it will not associate with the TAP in case of $n_{e}$ customers ahead. Therefore, PT2 is the best response against itself, i.e. a symmetric equilibrium.

To find the lower and upper bound of $n_{e}$, we propose Algorithm 1 to obtain $n_{l}$ and $n_{u}$.

```
Algorithm 1 Finding \(n_{l}\) and \(n_{u}\)
    Define \(g_{1}(n)\) and \(g_{2}(n)\) according to (18).
    Compute \(g_{1}(n)\) up to the first negative term;
    The highest equilibrium threshold \(n_{u}\) can be achieved;
    Compute \(g_{2}(n)\) starting from \(g_{2}\left(n_{u}+1\right)\) and going towards
    0 till the first positive term;
    The lowest equilibrium threshold \(n_{l}\) can be obtained.
    return \(n_{l}\) and \(n_{u}\).
```

3) Case 3. $I(t)$ is known and $N(t)$ is unknown
a) Queueing Analysis

When a customer with the access request is arriving at the queue of the TAP, it may not observe the queue length but
$I(t)$, i.e., whether the train is in the coverage of a TAP or not. In such a case, a mixed strategy that is specified by the joining probability $q(i) \in[0,1], i \in\{0,1\}$ is applied against the pure threshold strategies. In the following, $q(1)$ stands for the probability of associating when the train is in the the area of the TAP, and $q(0)$ is the probability of associating with the TAP when the train is in the area of cellular networks.

By denoting $\lambda_{i}=\lambda q(i), i \in\{0,1\}$, we can obtain the expected mean delay time and arrive at the following proposition,

Proposition 4. When all customers adopt a mixed strategies $(q(0), q(1))$, the expected mean delay time of a customer who enters the queue of the TAP and observes $I(t)=i$ can be given as

$$
\begin{align*}
& E(0, q(0), q(1))=\left(\frac{\xi \lambda_{0}+\mu \lambda_{0}+\theta \lambda_{1}-\lambda_{0} \lambda_{1}}{\mu \theta-\xi \lambda_{0}-\theta \lambda_{1}}+1\right) \frac{\theta+\xi}{\mu \theta}+\frac{1}{\theta}, \\
& E(1, q(0), q(1))=\left(\frac{\xi \lambda_{0}^{2}+\theta^{2} \lambda_{1}+\theta \xi \lambda_{0}}{\mu \theta^{2}-\theta \xi \lambda_{0}-\theta^{2} \lambda_{1}}+1\right) \frac{\theta+\xi}{\mu \theta} . \tag{21}
\end{align*}
$$

Proof: The proof is given in Appendix C.

## b) Equilibrium Strategy Analysis

With the above analysis on the delay time, we can obtain the equilibrium behavior in Theorem 3.

Theorem 3. An unique Nash equilibrium mixed strategy exist$s$ : when the customer is arriving, observe $I(t)$ and associate with the TAP with probability $q_{e}(I(t))$, where $\left(q_{e}(0), q_{e}(1)\right)$ is given by (22) where $q^{*}=\frac{\theta \mu(\mu-\lambda)\left(\varphi_{s} \theta-C\right)-(\theta+\xi) C \mu \theta}{\lambda(\mu-\lambda)(\theta+\xi) C+\xi \lambda \mu\left(\varphi_{s} \theta-C\right)}$.

Proof: The proof is given in Appendix D.
4) Case 4. Both $I(t)$ and $N(t)$ are unknown

## a) Queueing Analysis

In this case, the queue of the TAP is fully unobservable to the customer, i.e., both of $N(t)$ and $I(t)$ are unknown to the arriving customers. Since identical customers are considered, we assume each arriving customer joins with a probability $q$. Thus, the effective joining rate follows Poisson process with rate $\lambda^{\prime}=\lambda q$. Correspondingly, the expected mean delay time $E(D)$ can be derived as presented in Proposition 5.

Proposition 5. Considering the fact that all the customers associate with the TAP with the same strategy and with the same probability $q$, the expected mean delay time is expressed as

$$
\begin{equation*}
E(D)=\frac{\theta+\xi}{\mu \theta-\lambda^{\prime} \xi-\lambda^{\prime} \theta}+\frac{\mu \xi}{(\theta+\xi)\left(\mu \theta-\lambda^{\prime} \xi-\lambda^{\prime} \theta\right)} . \tag{23}
\end{equation*}
$$

Proof: We omit it here as the proof is similar to the one in Appendix C.

## b) Equilibrium Strategy Analysis

According to the expression of expected mean delay time, the unique Nash equilibrium mixed strategy can be obtained in Theorem 4.

Theorem 4. When $N(t)$ and $I(t)$ are fully unobservable, there exists an unique Nash equilibrium mixed strategy: the customer will associate with probability $q_{e}$, where $q_{e}$ is given

$$
\left(q_{e}(0), q_{e}(1)\right)= \begin{cases}(0,0), & \varphi_{s}<\frac{C(\theta+\xi)}{\theta \mu}  \tag{22}\\ \left(0, \frac{\mu \theta \varphi_{s}-C \theta-C \xi}{\varphi_{s} \theta \lambda}\right), & \frac{C(\theta+\xi)}{\theta \mu} \leq \varphi_{s} \leq \frac{C(\theta+\xi)}{\theta(\mu-\lambda)} \\ (0,1), & \frac{C(\theta+\xi)}{\theta(\mu-\lambda)}<\varphi_{s}<\frac{C(\theta+\xi)}{\theta(\mu-\lambda)}+\frac{C}{\theta} \\ \left(q^{*}, 1\right), & \frac{C(\theta+\xi)}{\theta(\mu-\lambda)}+\frac{C}{\theta} \leq \varphi_{s} \leq \frac{C\left(\mu \lambda-\lambda^{2}+\mu \theta\right)(\theta+\xi)}{\mu \theta(\mu \theta-\xi \lambda-\theta \lambda)}+\frac{C}{\theta} \\ (1,1), & \frac{C\left(\mu \lambda-\lambda^{2}+\mu \theta\right)(\theta+\xi)}{\mu \theta(\mu \theta-\xi \lambda-\theta \lambda)}+\frac{C}{\theta}<\varphi_{s}\end{cases}
$$

by

$$
q_{e}= \begin{cases}0, & \varphi_{s}<\frac{C(\theta+\xi)^{2}+C \mu \xi}{(\theta+\xi) \mu \theta}  \tag{24}\\ q_{e}^{*}, & \frac{C(\theta+\xi)^{2}+C \mu \xi}{(\theta+\xi) \mu \theta} \leq \varphi_{s} \leq \frac{C(\theta+\xi)^{2}+C \mu \xi}{(\theta+\xi)(\mu \theta-\lambda \xi-\lambda \theta)} \\ 1, & \frac{C(\theta+\xi)^{2}+C \mu \xi}{(\theta+\xi)(\mu \theta-\lambda \xi-\lambda \theta)}<\varphi_{s}\end{cases}
$$

where $q_{e}^{*}=\frac{\varphi_{s} \theta \mu(\xi+\theta)-C(\theta+\xi)^{2}-C \mu \xi}{(\theta+\xi)^{2} \lambda \varphi_{s}}$.
Proof: The proof is given in Appendix E.

## IV. Performance Evaluations and Discussions

In this section, analysis and simulation results are conduced to explore the impact of several parameters on the behavior of the customers. The system performance of different circumstances, i.e., when the state of the system are totally observed, partially observed, and totally unobserved. In addition, we also explore the performance of the expected mean delay time for each case and the association probability for Case 3 and Case 4, respectively. The analysis results in figures represent the expected mean delay time performance obtained from our association algorithm. From the figures in this section, the simulation results fitting the analytical results well show the accuracy of our proposed algorithm. Meanwhile, the proposed user association algorithm obtains much better delay performance compared with max signal to noise ratio (SNR) method.

## A. Performance of Case 1

For this case, it is assumed that both $I(t)$ and $N(t)$ are observable to the incoming customers. The expected mean delay time of each customer who enters the queue can be obtained by (3). In Fig. IV-A, we plot the expected mean delay time when the train is in the coverage of TAP or not with respect to $n$. From this figure, we can find that the expected delay time increases when more and more customers choose to wait in the queue. However, as $n$ increases to an extent, the incoming customers will abandon to join the queue of the TAP but stay in the cellular networks. Thus, the expected mean delay time maintains a constant value afterwards, which is consistent with PT1 in Case 1. In Fig. IV-A, the thresholds of the queue length for whether the customer should decide to stay in the queue can be found according to Theorem 1. From Fig. IV-A, we can find that the thresholds $n_{e}(I(t)), \forall I(t) \in\{0,1\}$ monotonically decrease with the distance between two adjacent TAPs. The larger the $\xi$ is, the more frequently the data transmission fails. Consequently, more customers decide not to associate with the TAPs. It can be also found that when the train is in the coverage of a TAP, the threshold is higher. In other words,


Fig. 2. Expected mean delay time vs. $n$ under Case 1 for $\lambda=0.5, \mu=5$, $\theta=0.2, \xi=1, \varphi_{s}=25, C=1$


Fig. 3. Thresholds vs. $\xi$ under Case 1 for $\lambda=0.5, \mu=5, \theta=0.2$, $\varphi_{s}=30, C=1$
when the train is in the coverage of a TAP, i.e., $I(t)=1$, the customer may tolerant a longer queue and prefer to associate. For example, when $\xi=0.4$ and the queue length is between 40 and 50 , if the train is within the coverage of a TAP, the customers with data requests prefer to associate with the TAP while they prefer not if the train is out of the transmission range of the TAP.


Fig. 4. Expected mean delay time vs. $n$ under Case 2 for $\lambda=0.5, \mu=5$, $\theta=0.2, \xi=1, \varphi_{s}=30, C=1$


Fig. 5. Thresholds vs. $\xi$ under Case 2 for $\lambda=0.5, \mu=5, \theta=0.2$, $\varphi_{s}=30, C=1$

## B. Performance of Case 2

We then investigate the performance of the second case, where the customers with data requests have no knowledge of $I(t)$. The expected mean delay time of each customer who enters the queue can be obtained from Proposition 2. Since Fig. IV-B presents the expected mean delay time varying with the number of customers in the queue when we have no knowledge of the accurate position of the train, it cannot figure out the expected mean delay time of two scenarios as in Fig. IV-A. Fig. IV-B only indicates the expected mean delay performance when the customers has knowledge of the number of customers in the queue. With the consideration of partial network state information, it is inevitable to allow redundant customers to join the queue in Case 2. Therefore, the expected mean delay time in Case 2 is longer than that in Case 1. In Fig. IV-B, we vary the value of $\xi$ and plot the thresholds (upper bound or lower bound) of the queue length and can observe that the thresholds ( $n_{u}$ and $n_{l}$ ) of the queue length


Fig. 6. Thresholds vs. $\varphi_{s}$ under Case 2 for $\lambda=0.5, \mu=5, \theta=0.2$, $\xi=1, C=1$


Fig. 7. Expected mean delay time vs. $\xi$ under Case 3 for $\lambda=0.5, \mu=5$, $\theta=0.2, \varphi_{s}=30, C=1$
are monotonically decreasing with $\xi$ when the time duration between two isolated TAPs increases. Since the customers can only access to the TAPs along the rail line for wireless access services, the increase in $\xi$ indicates that there will be a longer travel duration between two TAPs. As a result, an increasing $\xi$ may lead to frequent failures in data transmission. Correspondingly, the customers are reluctant to associate with the TAP and enter the queue of the TAP, and the queue length is therefore decreasing with the increasing of $\xi$. In Fig. IV-B, we plot the impact of $\varphi_{s}$ on the queue length thresholds. As we can see, the thresholds $n_{u}$ and $n_{l}$ increase with $\varphi_{s}$. This is mainly due to the fact that as reward $\varphi_{s}$ grows, the customers can afford higher cost of waiting in this queue. Therefore, more customers prefer to stay in the queue and thus the queue length increases.


Fig. 8. Expected mean delay time vs. $\theta$ under Case 3 for $\lambda=0.5, \mu=5$, $\xi=1, \varphi_{s}=30, C=1$

## C. Performance of Case 3

In the third case, an arriving customer with data requests is not informed about the queue length $N(t)$ but the status of TAP $I(t)$. In Fig. 7, we plot the expected mean delay time with respect to different values of $\xi$. We can also observe from this figure that the expected delay time increases with $\xi$. Similar to the observations in Fig. IV-B, we can see that the increase of $\xi$ may lead to an increased travelling duration between two TAPs. Such a phenomenon can lead to the problem that the customers are reluctant to stay in the queue if there exists frequent breakdown of the TAPs. In Fig. 8, the expected mean delay time is presented by varying the value of $\theta$. As we can see from Fig. 8, when the serving time $\theta$ of a TAP becomes larger, the expected mean delay time decreases. This is mainly due to the fact that when the TAPs are able to serve more data requests, the queue length can decrease, so as the delay time.

## D. Performance of Case 4

Here, we assume that the queue length and the state of the system are both fully unknown to the arriving customers, which means that $N(t)$ and $I(t)$ are unobservable. In Fig. 9, the relations between the expected mean delay time and data arrival rate $\lambda$ are presented. In general, it is shown that the expected mean delay time increases with the data arrival rate. This is due to the fact that as the data arrival rate grows, there will be more and more customers entering the queue. Correspondingly, the expected mean delay time increases. We can also observe from Fig. 9 that the expected mean delay time decreases with $\mu$. For example, when $\mu=1$ and $\lambda=0.35$, the expected delay is 12 . When $\mu=2$ and $\lambda=0.35$, the expected delay is 8 . This is mainly due to the reason that the TAP can serve the customer in a faster time when $\mu$ increases.

In Fig. 10, we vary the value of $\theta$ and describe the performance of the expected mean delay time with different $\xi$. From Fig. 10, we can observe that when $\theta$ increases, the mean delay time decreases dramatically when $\theta \leq 0.4$. The reason is that the waiting time of the customer is decreased by a


Fig. 9. Expected mean delay time vs. $\lambda$ under Case 4 for $\mu=5, \theta=0.2$, $\xi=1, \varphi_{s}=30, C=1$


Fig. 10. Expected mean delay time vs. $\theta$ under Case 4 for $\mu=5, \lambda=0.5$, $\xi=1, \varphi_{s}=30$ and $C=1$
longer service time. However, when the service time increases, i.e., $\theta>0.4$, more customers may decide to join the queue and associate with the TAPs for service. Correspondingly, the expected mean delay time does not show a significant change.

## E. User Association Probability Performance Evaluation

We further compare the association probability performance which can be observed in (22) and (24) from Fig. 11 and Fig. 12. From these two figures, we can see that the association probability of the fourth case is located between the third case when the train is in the coverage of TAP or not. Fig. 11 presents the association probabilities under Case 3 and Case 4 by varying the value of $\lambda$. We can see that the increase of the data arrival rate $\lambda$ may lead to the problem that the customers are not willing to enter the queue of the TAPs. This is because when more and more data are arrived, the TAP may be in a high load situation. It can also be found that the association probability when $I(t)=1$ is much higher than the


Fig. 11. Probability vs. $\lambda$ for $\mu=1, \theta=0.8, \xi=0.3, \varphi_{s}=4$ and $C=1$


Fig. 12. Probability vs. $\varphi_{s}$ for $\lambda=0.6, \mu=1, \theta=0.8, \xi=0.3$ and $C=1$
one when $I(t)=0$. In Fig. 12, the correspondingly association probabilities are presented with the value of $\varphi_{s}$ under Case 3 and Case 4. From Fig. 12, we can see that when the reward that the customer can obtain gets larger, the customers prefer to associate with the TAPs.

## V. Conclusion

In this paper, a series of TAPs which can provide highspeed data rates are randomly deployed along the railway. This kind of modern railway system is capable of supporting a wide range of on-board Internet services to satisfy the customers' demand. In order to efficiently allocate the network resources, we consider the user association problem for service provisioning in the heterogeneous wireless railway networks by applying a queueing game theoretic method. Particularly, we carry out the theoretical analysis on the expected mean delay time under the following circumstances: totally observed, partially observed, and totally unobserved the state of the system. According to the obtained delay value and the payoff
model, the customer's equilibrium strategies are proposed to decide whether to associate with a TAP or not. From the simulation results, the proposed service provisioning and user association scheme are verified that a unique Nash equilibrium mixed strategy exists in our heterogeneous wireless railway communication system. The simulation results matches well with the numerical values by our proposed algorithm for the expected mean delay time. The proposed user association algorithm obtains much better delay performance compared with max SNR method.

## Appendix A: Proof of Proposition 1

Based on the similar derivation in [21], we can find the $T(n, i)$ when both $n$ and $i$ are known. First, we assume $T(n, i)$ is the expected mean waiting time given that the customer finds the system at state $(n, i)$ just before arrival. Thus, for $n=1,2, \ldots$, we can have

$$
\begin{gather*}
T(n, 0)=\frac{1}{\theta}+T(n, 1)  \tag{25}\\
T(0,1)=\frac{1}{\mu+\theta}+\frac{\xi}{\mu+\xi} T(0,0)  \tag{26}\\
T(n, 1)=\frac{1}{\mu+\theta}+\frac{\mu}{\mu+\xi} T(n-1,1)+\frac{\mu}{\mu+\xi} T(n, 0) . \tag{27}
\end{gather*}
$$

When $n=0$, we have $T(0,0)$ and $T(0,1)$ in (25) and (26), respectively. By substituting (25) in (27), we obviously obtain a first-order recursive relation for $T(n, 1)$. Then, $T(n, 1)$ can be obtained in an iterative manner. We can also use (25) to obtain $T(n, 0)$. Therefore, we can arrive

$$
\begin{equation*}
T(n, i)=(n+1)\left(1+\frac{\xi}{\theta}\right) \frac{1}{\mu}+(1-i) \frac{1}{\xi} . \tag{28}
\end{equation*}
$$

## Appendix B: Proof of Proposition 2

The probability that a customer observes $n$ customers waiting in the queue and the train is not in the transmission range of TAP can be expressed as

$$
\begin{equation*}
P_{0}=\frac{\lambda p(n, 0)}{\lambda p(n, 0)+\lambda p(n, 1)} \tag{29}
\end{equation*}
$$

As presented in [21], $E(n)$ can be given as

$$
\begin{equation*}
E(n)=E(n, 1)+\frac{1}{\theta} P_{0} \tag{30}
\end{equation*}
$$

where $E(n, 1)$ is the expected delay time given that the customer finds $N(t)=n, I(t)=1$ upon arrival. The expected delay time of an arriving customer who finds the system is at $N(t)=n, I(t)=i$ and decides to associate with the TAP can be given as [21]

$$
\begin{equation*}
E(n, i)=(n+1)\left(1+\frac{\xi}{\theta}\right) \frac{1}{\mu}+(1-i) \frac{1}{\theta} \tag{31}
\end{equation*}
$$

Substituting (31) into (30) with $i=1$, and using the stationary probability in Proposition 3 to obtain (29), Proposition 2 is able to be proved by (30).

## Appendix C: Proof of Proposition 4

Similar to the proof of Proposition 2, $p(n, i)$ can be found by using the balance equations,

```
C1: \(\left(\lambda_{0}+\theta\right) p(0,0)=\xi p(0,1)\),
C2: \(\left(\lambda_{0}+\theta\right) p(n, 0)=\lambda_{0} p(n-1,0)+\xi p(n, 1)\),
C3: \(\left(\lambda_{1}+\xi\right) p(0,1)=\mu p(1,1)+\theta p(0,0)\),
C4: \(\left(\lambda_{1}+\mu+\xi\right) p(n, 1)\)
    \(=\mu p(n+1,1)+\theta p(n, 0)+\lambda_{1} p(n-1,1)\),
C5: \(\mu p(n+1,1)=\lambda_{0} p(n, 0)+\lambda_{1} p(n, 1)\).
```

To obtain the delay time, the probability generating function $G_{i}(z)=\sum_{k=0}^{\infty} z^{k} p(n, i), i \in\{0,1\},|z| \leq 1$ can be used. By Multiplying $z^{k}$ with both sides of $\mathbf{C 1}, \mathbf{C 2}$ and $\mathbf{C 5}$ and summing over all $k$, one can have

$$
\begin{gather*}
\left(\lambda_{0}+\theta\right) G_{0}(z)=\xi G_{1}(z)+\lambda_{0} z G_{0}(z)  \tag{33}\\
\mu\left(G_{1}(z)-p(0,1)\right)=\lambda_{0} G_{0}(z) z+\lambda_{1} G_{0}(z) z \tag{34}
\end{gather*}
$$

With $z=1$ and the help of normalizing equation and $\lim _{z \rightarrow 1} G_{0}(z)+\lim _{z \rightarrow 1} G_{1}(z)=1$, we have

$$
\begin{align*}
G_{0}(1) & =\frac{\xi}{\theta+\xi} \\
G_{1}(1) & =\frac{\theta}{\theta+\xi} \tag{35}
\end{align*}
$$

Substituting (33) into (34), we have

$$
\begin{equation*}
\mu\left[G_{0}(z)-p(0,1)\right]=\left(\lambda_{0}+\theta\right) G_{0}(z)-\xi G_{1}(z)+\lambda_{1} z G_{1}(z) \tag{36}
\end{equation*}
$$

Differentiating (33) and (36) over $z$, we can obtain

$$
\begin{align*}
G_{0}^{\prime}(z) & =\frac{\lambda_{0} G_{0}(z)+\xi G_{1}^{\prime}(z)}{\lambda_{0}+\theta-\lambda_{0} z}  \tag{37}\\
G_{1}^{\prime}(z) & =\frac{\left(\lambda_{0}+\theta\right) G_{0}^{\prime}(z)+\lambda_{1} G_{1}(z)}{\mu-\lambda_{1}+\xi} .
\end{align*}
$$

Letting $z=1$, we can arrive at

$$
\begin{align*}
G_{0}^{\prime}(1) & =\frac{\lambda_{0}\left(\xi^{2}+\mu\right)+\lambda_{1}\left(\theta-\lambda_{0}\right)}{(\xi+\theta)\left(\mu \theta-\xi \lambda_{0}-\theta \lambda_{1}\right)} \\
G_{1}^{\prime}(1) & =\frac{\xi \lambda_{0}^{2}+\lambda_{1} \theta^{2}+\theta \xi \lambda_{0}}{(\xi+\theta)\left(\mu \theta-\xi \lambda_{0}-\theta \lambda_{1}\right)} \tag{38}
\end{align*}
$$

Due to the PASTA property [16], when $I(t)=i$, the probability that the arrival customer observes $n$ customers waiting in the queue of the TAP can be given as

$$
\begin{equation*}
\phi(n, i)=\frac{p(n, i)}{\sum_{j} p(j, i)}, n \in \mathbb{N}^{0} . \tag{39}
\end{equation*}
$$

Denoting that $E(N \mid I=i)$ as the expected number of customers in the queue when $I(t)=i$, we have

$$
\begin{equation*}
E(N \mid I=i)=\sum_{n=i}^{+\infty} n \phi(n, i)=\frac{G_{i}(1)}{G_{i}^{\prime}(1)} \tag{40}
\end{equation*}
$$

Then substituting (40) into (31), we have
$E(i, q(0), q(1))(E(N \mid I=i)+1)\left(1+\frac{\xi}{\theta}\right) \frac{1}{\mu}+(1-i) \frac{1}{\theta}$.
Proposition 4 has been proved.

## Appendix D: Proof of Theorem 3

Based on the payoff model in (2), we denote the expected payoff accordingly as

$$
\begin{align*}
& R(0, q(0), q(1))=\varphi_{s}-C E(0, q(0), q(1)),  \tag{42}\\
& R(1, q(0), q(1))=\varphi_{s}-C E(1, q(0), q(1)) .
\end{align*}
$$

Therefore, when $I(t)=i$, a customer with the data request will associate with the TAP if $R(i, q(0), q(1))>0$, balk if $R(i, q(0), q(1))<0$. The customer is indifferent if $R(i, q(0), q(1))=0$. Correspondingly, five cases are considered here as follows:

1) If $\frac{\varphi_{s}}{C}<\frac{\xi+\theta}{\theta \mu}$, then we have $R(1,0,0)<0$. According to (21) and (42), we can see that $R(1, q(0), q(1))$ is strictly decreasing for $q(0) \in[0,1]$ and $q(1) \in[0,1]$, respectively. We can see that $R(1, q(0), q(1)) \leq R(1,0,0)<0$. Therefore, if $(q(0), q(1))$ is the strategy for all other customers, a negative payoff is received by the considered customer from associating. Therefore, the association probability of the customer is 0 when $I(t)=1$. We can also see that when $I(t)=0$, the association probability is 0 as well. Thus, $\frac{\varphi_{s}}{C}<\frac{\xi+\theta}{\theta \mu},(q(0), q(1))=(0,0)$.
2) If $\frac{\theta+\xi}{\theta(\mu-\lambda)}<\frac{\varphi_{s}}{C}<\frac{\theta+\xi}{\theta(\mu-\lambda)}+\frac{1}{\theta}$, we have $R(1,0,1)>0$ and $R(0,0,1)<0$. We can also find that $R(1,0, q(1))$ is strictly decreasing for $q(1) \in[0,1]$, and $R(1,0, q(1)) \geq$ $R(1,0,1)>0$. Thus, if $(q(0)=0, q(1) \leq 1)$ is the strategy that all other customers use, the considered customer can obtain a non-negative payoff from associating. Therefore, if $I(t)=1$, the best response is 1. Meanwhile, as $R(0, q(0), 1)$ is strictly decreasing for $q(0) \in[0,1]$. Therefore, following the previous analysis, we can see that $q_{e}(0)=0$.
3) If $\frac{\theta+\xi}{\theta \mu} \leq \frac{\varphi_{s}}{C} \leq \frac{\theta+\xi}{\theta(\mu-\lambda)}$, we have $R(1,0,1) \leq 0 \leq$ $R(1,0,0)$. As what we have in Cases 1 and 2, and $q_{e}(0)$ is non-decreasing of $R$, we can then obtain $q_{e}(0)=0$. Correspondingly, we can see that $R(1,0, q(1))$ is strictly decreasing for $q(1) \in[0,1]$. Thus, if all the customers who find $I(t)=1$ associate with probability $q_{e}(1)=1$, then $R(1,0,1) \leq 0$, and $q_{e}(1)=1$ does not result in an equilibrium. In addition, if $q_{e}(1)=0$ is used as the strategy for all other customers, $R(1,0,0) \geq 0$, and thus, the considered customer obtains a non-negative payoff from associating with the TAP, which means that the best response is 1 . Therefore, neither $q_{e}(1)=0$ nor $q_{e}(1)=$ 1 is not an equilibrium mixed strategy. Hence, $q^{\prime} \in[0,1]$ can be used as the equilibrium mixed strategy such that $R\left(1,0, q^{\prime}\right)=0$ and $q$ can be obtained accordingly.
4) If $\frac{\left(\mu \lambda-\lambda^{2}+\mu \theta\right)(\theta+\xi)}{\mu \theta(\mu \theta-\xi \lambda-\theta \lambda)}+\frac{1}{\theta}<\frac{\varphi_{s}}{C}$, we can see $R(1,1,1)>0$. It can also be verified that $R(1, q(0), q(1))$ is strictly decreasing for $q(0) \in[0,1]$ and for $q(1) \in[0,1]$. So, if the other customers use $(q(0), q(1))$ as the strategy, the considered customer has positive payoff when it finds $I(t)=1$ and joins the queue. Thus, the best response is $q_{e}(1)=1$. It can also be found that $R(0,1,1)>0$ and $R(0, q(0), q(1))$ is strictly decreasing for $q(0) \in[0,1]$, and we have $0<R(0,1,1) \leq R(0, q(0), q(1))$. Thus, if all other customers use $q_{e}(0), 1$ as the strategy, the considered customer has a positive payoff from entering
so he can enter with probability 1 when observing $I(t)=0$.
5) If $\frac{\theta+\xi}{\theta(\mu-\lambda)}+\frac{1}{\theta} \leq \frac{\varphi_{s}}{C} \leq \frac{\left(\mu \lambda-\lambda^{2}+\mu \theta\right)(\theta+\xi)}{\mu \theta(\mu \theta-\xi \lambda-\theta \lambda)}+\frac{1}{\theta}$, we have $R(0,1,1) \leq 0 \leq R(0,0,1)$. From the observations from Cases 2 and 4 , and $q_{e}(1)$ is non-decreasing of $\varphi_{s}$, we can obtain $q_{e}(1)=1$. Similar to the analysis in Case 3, the equilibrium mixed strategy $q_{e}(0)$ can be obtained by addressing $R(0, q(0), 1)=0$.

## Appendix E: Proof of Theorem 4

Based on the payoff model in (2), if a customer decides to associate with the TAP, the expected payoff is

$$
\begin{equation*}
U(q)=\varphi_{s}-C E(D) \tag{43}
\end{equation*}
$$

where $E(D)$ is given in (23). It can be noticed that $U(q)$ is a strictly decreasing function for $q \in[0,1]$ and

$$
\begin{gather*}
U(0)=\varphi_{s}-C \frac{(\theta+\xi)^{2}+\mu \xi}{(\theta+\xi) \mu \theta}  \tag{44}\\
U(1)=\varphi_{s}-C \frac{(\theta+\xi)^{2}+\mu \xi}{(\theta+\xi)(\mu \theta-\lambda \theta-\lambda \xi-\lambda \theta)} \tag{45}
\end{gather*}
$$

When $\frac{(\theta+\xi)^{2}+\mu \xi}{(\theta+\xi) \mu \theta} \leq \frac{\varphi_{s}}{C} \leq \frac{(\theta+\xi)^{2}+\mu \xi}{(\theta+\xi)(\mu \theta-\lambda \xi-\lambda \theta)}$, there exists a unique solution $q_{e}^{*}$ of the equation $U(q)=0$ which lies in the interval $[0,1]$. When $\frac{\varphi_{s}}{C}<\frac{(\theta+\xi)^{2}+\mu \xi}{(\theta+\xi) \mu \theta}$, the customer will suffer a negative benefit if he associates with the TAP. In this case, the best response is 0 . Similarly, when $\frac{(\theta+\xi)^{2}+\mu \xi}{(\theta+\xi)(\mu \theta-\lambda \xi-\lambda \theta)}<\frac{\varphi_{s}}{C}$, we can obtain that $U(q)>0, \forall q \in[0,1]$. And the customer's best response is 1 .

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Yun Hu received the M.S. degree in Electronics and
 communications engineering from Xidian University, Xi'an, China, in 2013. She is currently working toward the $\mathrm{Ph} . \mathrm{D}$. degree in military communications with the State Key Lab of ISN, Xidian University, Xi’an, China. From Oct. 2014 to Apr. 2016, she was also with Houston University, Houston, Texas, USA, as a visiting scholar funded by China Scholarship Council (CSC). Her current research interests include high-speed train networks, vehicular ad hoc networks, stochastic network calculus and martingale theory analysis.


Zheng Chang (M'13) received the B.Eng. degree from Jilin University, Changchun, China in 2007, M.Sc. (Tech.) degree from Helsinki University of Technology (Now Aalto University), Espoo, Finland in 2009 and Ph.D degree from the University of Jyväskylä, Jyväskylä, Finland in 2013. Since 2008, he has held various research positions at Helsinki University of Technology, University of Jyväskylä and Magister Solutions Ltd in Finland. He was a visiting researcher at Tsinghua University, China, from June to August in 2013, and at University of Houston, TX, from April to May in 2015. He has been awarded by the Ulla Tuominen Foundation, the Nokia Foundation and the Riitta and Jorma J. Takanen Foundation for his research work. Currently he is working with University of Jyväskylä and his research interests include radio resource allocation, Internet of Things, cloud computing and green communications.


Hongyan Li (M’08) received the M.S. degree in control engineering from Xi'an Jiaotong University, Xi'an, China, in 1991 and the Ph.D. degree in signal and information processing from Xidian University, Xi'an, in 2000. She is currently a Professor with the State Key Laboratory of Integrated Service Networks, Xidian University. Her research interests include wireless networking, cognitive networks, integration of heterogeneous network, and mobile ad hoc networks.


Tapani Ristaniemi(SM'11) received his M.Sc. in 1995 (Mathematics), Ph.Lic. in 1997 (Applied Mathematics) and Ph.D. in 2000 (Wireless Communications), all from the University of Jyväskylä, Jyväskylä, Finland. In 2001 he was appointed as Professor in the Department of Mathematical Information Technology, University of Jyväskylä. In 2004 he moved to the Department of Communications Engineering, Tampere University of Technology, Tampere, Finland, where he was appointed as Professor in Wireless Communications. In 2006 he moved back to University of Jyväskylä to take up his appointment as Professor in Computer Science. He is an Adjunct Professor of Tampere University of Technology. In 2013 he was a Visiting Professor in the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore.
He has authored or co-authored over 150 publications in journals, conference proceedings and invited sessions. He served as a Guest Editor of IEEE Wireless Communications in 2011 and currently he is an Editorial Board Member of Wireless Networks and International Journal of Communication Systems. His research interests are in the areas of brain and communication signal processing and wireless communication systems research.

Besides academic activities, Professor Ristaniemi is also active in the industry. In 2005 he co-founded a start-up Magister Solutions Ltd in Finland, specialized in wireless system R\& D for telecom and space industries in Europe. Currently he serves as a consultant and a Member of the Board of Directors.


Zhu Han (S'01-M'04-SM'09-F'14) received the B.S. degree in electronic engineering from Tsinghua University, in 1997, and the M.S. and Ph.D. degrees in electrical and computer engineering from the University of Maryland, College Park, in 1999 and 2003, respectively.
From 2000 to 2002, he was an R\&D Engineer of JDSU, Germantown, Maryland. From 2003 to 2006, he was a Research Associate at the University of Maryland. From 2006 to 2008, he was an assistant professor at Boise State University, Idaho. Currently, he is a Professor in the Electrical and Computer Engineering Department as well as in the Computer Science Department at the University of Houston, Texas. His research interests include wireless resource allocation and management, wireless communications and networking, game theory, big data analysis, security, and smart grid. Dr. Han received an NSF Career Award in 2010, the Fred W. Ellersick Prize of the IEEE Communication Society in 2011, the EURASIP Best Paper Award for the Journal on Advances in Signal Processing in 2015, IEEE Leonard G. Abraham Prize in the field of Communications Systems (best paper award in IEEE JSAC) in 2016, and several best paper awards in IEEE conferences. Currently, Dr. Han is an IEEE Communications Society Distinguished Lecturer.


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    Y. Hu and $\mathrm{H} . \mathrm{Li}$ are with the State Key Laboratory of Integrated Service Networks, Xidian University, Xi'an 710071, China (e-mail: yhu_1@stu.xidian.edu.cn, hyli@xidian.edu.cn).
    Z. Chang and T. Ristaniemi are with Department of Mathematical Information Technology, University of Jyvaskyla, FI-40014 Jyvaskyla, Finland (email: zheng.chang@jyu.fi, tapani.ristaniemi@jyu.fi).
    Z. Han is with Department of Electrical and Computer Engineering, Joint Appointment with Computer Science Department, University of Houston, Houston, Texas, USA (e-mail: zhan2@uh.edu).

[^1]:    ${ }^{1}$ In order to lease the load of the cellular networks and decrease frequent handovers between TAPs and BSs, we study the user-TAP association problem in the heterogeneous railway network by fully utilizing the TAPs for mobile users.

[^2]:    ${ }^{2}$ In economic field, risk neutral is a mindset concept where an investor is indifferent to risk when making an investment decision. If the payoff is positive, the customers prefer to choose to associate with the TAP. If the payoff is negative, the customers decide not to join in the queue of the TAP. If the payoff is equal to 0 , the customers are indifferent to whether to associate with the TAP or not. In this sense, we consider that the customers are risk neutral.

[^3]:    ${ }^{3}$ For providing full consideration of each aspect, in this scenario, we consider that there are deployed some mobile relays on the train, which the relays can be connected to the TAPs along the railway. Thus, it is possible for the customers to know $N(t)$ if he/she is not within a coverage of a TAP.

