Generalization of Linked Tensor Decomposition for Group Analysis
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Introduction

background
- Linked tensor decomposition can be used to extract common components, individual components and core tensors simultaneously.
- Linked tensor decomposition can effectively utilize linking/coupling information to improve the identifiability of decomposition.
- Linked tensor decomposition has its advantages in imposing constraints, which can contribute to obtaining more reasonable decomposition results with convincing physiological or pathological interpretations.

The purpose of the proposed algorithm
To develop a more generalized and flexible model with inconsistent component number of linked tensor decomposition.

Proposed algorithm

Linked tensor decomposition of CP model
- LCPTD model: Consistent component number, as in Fig.1(a).
- Generalization of LCPTD (GLCPTD): Inconsistent component number, as in Fig. 1(b).
- Each factor matrix \( U^{(n,s)} = [u_i^{(n,s)}] \) consists of two parts: \( U_c^{(n,s)} \in \mathbb{R}^{a,n \times b} \) or \( U_l \) shared by all tensor blocks with coupling information and \( U_i^{(n,s)} \in \mathbb{R}^{a,n \times (R-L_n)} \), \( 0 \leq L_n \leq R \) representing individual characteristics of each single tensor block.

Realization of GLCPTD
- Euclidean Deconvolution minimization
- Hierarchical Alternating Least Squares (HALS)
- The cost function can be expressed as:

\[
\begin{align*}
\text{minimize} & \quad \sum_{s=1}^{S} \left\| X_s - \sum_{r=1}^{R(s)} \lambda_r^{(s)} u_r^{(1,s)} \circ u_r^{(2,s)} \circ \cdots \circ u_r^{(N,s)} \right\|_F^2 \\
\text{s.t.} & \quad u_r^{(n,s)} = \cdots = u_r^{(N,s)} \quad \forall r \leq L_n, \\
& \quad \left\| u_r^{(n,s)} \right\|_F = 1, n = 1 \ldots N, r = 1 \ldots R(s), s = 1 \ldots S
\end{align*}
\]

Experiment

Image reconstruction and denoising
- Yale face database
- 165 gray-scale images of 15 individuals
- 11 images per subject with different facial expressions
- Noise: 5% Salt-and-pepper noise
- Component number: LCPTD (36), GLCPTD (PCA-99.6%)
- Construction of tensors: (1) Face images from the same subject with different expressions; (2) Face images from different subjects with the same expression.

Results

Table 1 shows the averaged PSNRs of reconstructed images under two conditions. Fig. 3 and Fig. 4 depict the original, noisy and reconstructed face images from one subject with four random expressions (‘centerlight’, ‘glasses’, ‘happy’ and ‘leftlight’) respectively.

![Fig. 2. PSNRs of noisy and reconstructed face images.](image)

Table 1. Averaged PSNRs (dB) of reconstructed images of condition I&II

<table>
<thead>
<tr>
<th>Condition I</th>
<th>Condition II</th>
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<tr>
<td>(L_1, L_2)</td>
<td>10 20 30</td>
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Conclusion

The main objective of this paper is to develop a generalized and flexible model of linked tensor decomposition which is more suitable for group analysis. The results of image reconstruction and denoising illustrate the superior performance of the newly generalized model. However, the selection of parameter \( L_n \) is still an open issue in the current study, which will be one of our future works.

References


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