

Lecture 12: Bose-Einstein Condensation

1 Introduction

Bose-Einstein condensation is a quantum phenomenon in Bose gases in which a large number of bosons simultaneously occupy the ground state of a system. Bose-Einstein condensates were predicted in 1925 by Bose and discovered experimentally 70 years later by Weimann, Cornell and Ketterle who shared the Nobel prize for their discovery in 2001.

Your first thought might be, of course a lot of bosons are in the ground state! After all, there is no quantum effect preventing them from being in the ground state (no Pauli exclusion), and naturally particles want to be in the state of lowest energy. This is a good thought, and I'm glad you had it. Let's follow through. How many bosons do you expect in the ground state? Well, say they obey Maxwell-Boltzmann statistics, so that $n_i \sim e^{-\varepsilon_i/k_B T}$. This function is pretty flat for $\varepsilon_i \ll k_B T$, so we would expect that if there are say 100 states below $k_B T$ then each one should have roughly the same number of particles in it – nothing too special about the ground state. Thus, if you want a sizable fraction, say 1/2 the particles, to be in the ground state, you would have to get $k_B T$ down below the energy of the first excited state ε_1 . This argument is correct, and we'll flesh it out more in a moment. The amazing thing is that with Bose-Einstein statistics this isn't true – you can find more than half of the particles in the ground state even for temperatures with $k_B T \gg \varepsilon_1$.

Bose-Einstein condensation is tricky to explain, so we'll approach it different ways. First, we'll try to understand what it is about Bose-Einstein statistics that allows condensation to happen through a simple system that we can solve in the canonical ensemble. Then we'll do the general case using the grand-canonical ensemble, first numerically, and then through analytic expansions.

In this lecture, it will be helpful to set the ground state energy to zero: $\varepsilon_0 = 0$. By setting it to zero, we mean that we list all energies as *relative* to the ground state. Shifting all the energies as well as the chemical potential in this way will have no effect on the physics and can be done without loss of generality.

2 Two-state system: canonical ensemble

Consider a system with N particles but only two possible energy states: $\varepsilon_0 = 0$ and $\varepsilon_1 = \varepsilon$. Because there are only two states, we can study this system in the canonical ensemble for both Maxwell-Boltzmann statistics and Bose-Einstein statistics. We'll see that even in this two-state system the ground state occupancy can be much larger than $\frac{N}{2}$ even when $k_B T \gg \varepsilon$ with Bose-Einstein statistics. With the large number of states present in any realistic system, the canonical ensemble will not be tractable and we will have to resort to the more abstract grand canonical ensemble.

Let's start with the simplest case of our 2-state system $N = 1$. If there is only one particle, then for any statistics, the partition function is

$$Z_1 = \sum_k e^{-\beta E_k} = 1 + e^{-\beta \varepsilon} \quad (1)$$

The probability of finding the particle in the ground state is $P_{\text{ground}} = \frac{1}{Z_1} e^{-\beta \varepsilon_0} = \frac{1}{Z_1}$ and so the expected fraction of particles in the ground state is

$$\frac{\langle N_{\text{ground}} \rangle}{N} = 1 \cdot P_{\text{ground}} + 0 \cdot P_{\text{not-ground}} = \frac{1}{Z_1} = \frac{1}{1 + e^{-\beta \varepsilon}} \quad (2)$$

Again, this holds for any statistics, since there is only one particle.

Now say there are N particles. With Maxwell-Boltzmann statistics, the probability of finding any particle in the ground state is independent of the probability of finding any other particle anywhere. This implies that the N particle partition function is related to the 1 particle one by

$$Z_N^{\text{MB}} = \frac{1}{N!} (Z_1)^N = \frac{1}{N!} (1 + e^{-\beta\epsilon})^N \quad (3)$$

Of the 2^N microstates, there is only one microstate with all the particles in the ground state, and we divide by $N!$ since the particles are identical. Thus

$$P_{\text{allground}} = \frac{1}{N!} \frac{1}{Z_N^{\text{MB}}} e^{-\beta\epsilon_0} = \frac{1}{N!} \frac{1}{Z_N^{\text{MB}}} \quad (4)$$

There are N states with 1 particle in the excited state, $\binom{N}{2}$ states with 2 particles in the excited state and so on. As a check on this the probabilities should sum to one:

$$\sum_k P_k = \frac{1}{N! Z_N^{\text{MB}}} \left[1 + N e^{-\beta\epsilon} + \binom{N}{2} e^{-2\beta\epsilon} + \dots + \binom{N}{N} e^{-N\beta\epsilon} \right] = \frac{1}{N! Z_N^{\text{MB}}} (1 + e^{-\beta\epsilon})^N = 1 \quad (5)$$

where Eqs. (1) and (3) were used in the last step. To compute the expected number in the ground state, we multiply each term in this sum by the ground state occupancy:

$$\langle N_{\text{ground}}^{\text{MB}} \rangle = \frac{1}{N! Z_N^{\text{MB}}} \left(N \cdot 1 + (N-1) \cdot N e^{-\beta\epsilon} + (N-2) \cdot \binom{N}{2} e^{-2\beta\epsilon} + \dots + 0 \cdot \binom{N}{N} e^{-N\beta\epsilon} \right) \quad (6)$$

$$= \frac{N}{1 + e^{-\epsilon\beta}} \quad (7)$$

To do the sum, I used Mathematica, but you can do it by hand without too much work. Note that at large T ($\beta \rightarrow 0$), $\langle N_{\text{ground}}^{\text{MB}} \rangle$ goes to $\frac{N}{2}$: half the particles are in the ground state, half in the excited state.

With Bose-Einstein statistics there is only one state with m particles in the ground state and $N - m$ particles in the excited state. So there are only $N + 1$ possible states all together and

$$Z_N^{\text{BE}} = 1 + e^{-\beta\epsilon} + e^{-2\beta\epsilon} + \dots + e^{-N\beta\epsilon} = \frac{1 - e^{-(N+1)\beta\epsilon}}{1 - e^{-\beta\epsilon}} \quad (8)$$

Then the expected number in the ground state is

$$\langle N_{\text{ground}}^{\text{BE}} \rangle = \frac{1}{Z_N^{\text{BE}}} [N \cdot 1 + (N-1) \cdot e^{-\beta\epsilon} + (N-2) \cdot e^{-2\beta\epsilon} + \dots + 0 \cdot e^{-N\beta\epsilon}] \quad (9)$$

$$= \frac{1}{e^{-\beta\epsilon} - 1} + \frac{N+1}{1 - e^{-(N+1)\beta\epsilon}} \quad (10)$$

Let us look at Eqs. (7) and (10) numerically for $N = 100$:

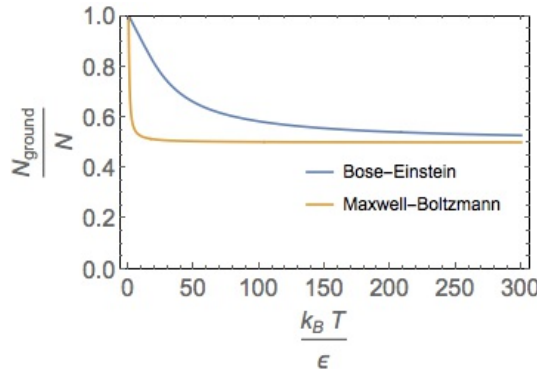


Figure 1. The fractional population of the ground state in a two state system with $N = 100$.

This plot demonstrates Bose-Einstein condensation. With Maxwell-Boltzmann statistics, the temperature has to be very low to get the lowest state to have an appreciable filling fraction. At temperature $k_B T \gtrsim \varepsilon$ both the ground state and the first excited state are around equally populated so $\langle N_{\text{ground}} \rangle = \frac{N}{2}$. In contrast, with Bose-Einstein statistics, a significant fraction of the particles are in the ground state even well above the temperature $k_B T = \varepsilon$. For example, with at $k_B T = \frac{N}{10} \varepsilon \gg \varepsilon$ we find 90% of the atoms are in the ground state for Bose-Einstein statistics, but only 52% for Maxwell-Boltzmann statistics. This demonstrates Bose-Einstein condensation.

Bose-Einstein condensation is a phase transition whereby the ground state become highly occupied. What is the critical temperature for this to occur? There is only two dimensionless numbers we can work with $\frac{k_B T}{\varepsilon}$ and N . We want N to be large, $N \gg 1$. Then we can expand in two limits $\frac{k_B T}{\varepsilon} \gg N$ (i.e. $\beta \varepsilon N \ll 1$) and $\frac{k_B T}{\varepsilon} \ll N$ (i.e. $\beta \varepsilon N \gg 1$). Expanding Eq. (10) in the first limit gives

$$\frac{\langle N_{\text{ground}}^{\text{BE}} \rangle}{N} = \frac{1}{2} + \frac{N\varepsilon}{12k_B T} + \dots \quad \left(\frac{k_B T}{\varepsilon} \gg N \right) \quad (11)$$

This is the true high-temperature limit, where the classical behavior $\frac{\langle N_{\text{ground}}^{\text{BE}} \rangle}{N} \rightarrow \frac{\langle N_{\text{ground}}^{\text{MB}} \rangle}{N} \approx \frac{1}{2}$ is approached as $T \rightarrow \infty$. In the second limit $N \gg \frac{k_B T}{\varepsilon} \gg 1$, the temperature is large, but not too large, and the expansion gives a different result

$$\frac{\langle N_{\text{ground}}^{\text{BE}} \rangle}{N} = 1 - \frac{k_B T}{N\varepsilon} + \dots \quad \left(N \gg \frac{k_B T}{\varepsilon} \right) \quad (12)$$

This limit shows the growth of $\frac{\langle N_{\text{ground}}^{\text{BE}} \rangle}{N}$ toward 1 as $T \rightarrow 0$. The comparison of these approximations to Eq. (10) looks like

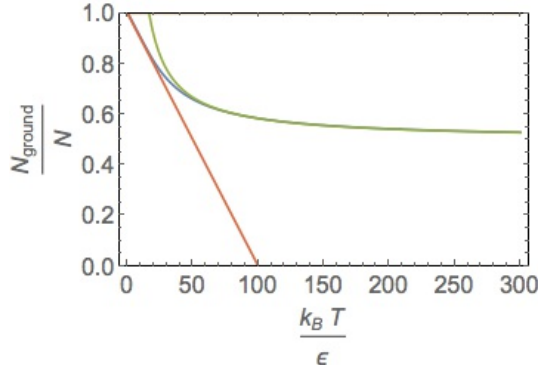


Figure 2. The approximations to the Bose-Einstein curve in Eqs. (11) and (12).

The crossover point is roughly where the approximations used for our expansions break down. The first term is the same order as the second term in Eq. (11) when $k_B T = \frac{N\varepsilon}{6}$. For Eq. (12) the crossover is at $k_B T = N\varepsilon$. Thus we find a critical temperature $T_c \sim \frac{N\varepsilon}{6k_B} \sim \frac{N\varepsilon}{k_B}$ for this two state model indicating the onset of Bose-Einstein condensation. (The crossover point in this 2-state example is not at a precise temperature, as you can see from the plot. T_C will become precise when we consider a realistic system with a large number of states in the next section.)

To emphasize how strange Bose-Einstein condensation is, remember that at $k_B T \gg \varepsilon$ we should be able to use $e^{-\beta\varepsilon} \approx 1$ independent of N . That is, the ground state and first excited state should have pretty similar thermodynamic properties and occupancy at high temperature. This is *not* what we are finding. Instead, for $N = 10$ million, with a temperature 1 million times the excited state energy, 90% of the atoms are in the ground state and only 10% are in the excited state.

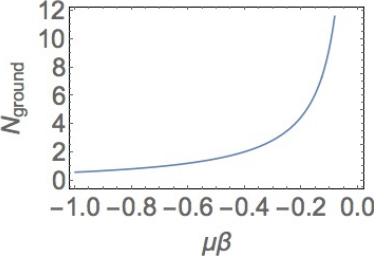
Now, this 2-state model is not a realistic approximation to any physical system. It turns out to be very difficult to calculate $\langle N_{\text{ground}}^{\text{BE}} \rangle$ in the canonical ensemble for a realistic system that has an infinite number of states. The difficulty is that we have to count the number of ways of allocating N particles to the states, and then to perform the sum over occupancies of the ground state times Boltzmann factors. It turns out to be much easier to compute the general case using the grand-canonical ensemble with μ instead of N , as we will now see.

3 Grand canonical ensemble

With Bose-Einstein statistics, we determined that using the grand canonical ensemble the expected number of particles in a state i is

$$\langle n_i \rangle = \frac{1}{e^{\beta(\varepsilon_i - \mu)} - 1} \quad (13)$$

with ε_i the energy of the state i . So the expected number of particles in the ground state ($\varepsilon_i = 0$) is

$$\langle N_{\text{ground}} \rangle = \frac{1}{e^{-\beta\mu} - 1} = N_{\text{ground}} \quad (14)$$


Recall that for Bose gases, μ will always be lower than all the energies (i.e. $\mu < 0$ when $\varepsilon_0 = 0$). We can see this explicitly from the plot.

We can invert Eq. (14) to solve for μ in terms of $\langle N_{\text{ground}} \rangle$:

$$e^{-\beta\mu} = \frac{1}{\langle N_{\text{ground}} \rangle} + 1 \quad (15)$$

Our strategy will then be to find the ground state occupancy by using the constraint the the total number of particles is N . That is, we will compute

$$N = \sum_i \langle n_i \rangle = \sum_i \frac{1}{e^{\beta\varepsilon_i} e^{-\beta\mu} - 1} = \sum_i \frac{1}{e^{\beta\varepsilon_i} \left(\frac{1}{\langle N_{\text{ground}} \rangle} + 1 \right) - 1} \quad (16)$$

Once we work out the ε_i we can do the sum numerically or analytically and therefore find $\frac{\langle N_{\text{ground}} \rangle}{N}$.

Before beginning the calculation, let us quickly ask about the non-ground state occupancies. Can they be large too? Note that for the ground state, the occupancy is large when $\mu \approx 0 = \varepsilon_0$: it is in this limit that $\langle N_{\text{ground}} \rangle$ blows up. For excited states, we would need $\mu \approx \varepsilon_i$. However, μ has an upper bound of $\varepsilon_0 = 0$, so μ cannot get arbitrarily close to ε_i and therefore condensation cannot happen in any excited state. The ground state is special.

3.1 Exact numerical solution

Now let us calculate N , and hence $\frac{\langle N_{\text{ground}} \rangle}{N}$. Bose-Einstein condensation is relevant at low temperature, where particles are non-relativistic. So consider a non-relativistic gas of monatomic bosonic atoms in a 3D box of size L . The allowed wavevectors of the system are $\vec{k}_n = \frac{\pi}{L} \vec{n}$ just like for photons or phonons, and the momenta are $\vec{p} = \hbar \vec{k}$ as always. In a non relativistic system, the energies are

$$\varepsilon_n = \frac{\hbar^2 \vec{k}_n^2}{2m} = \frac{\hbar^2 \pi^2}{2m L^2} \vec{n}^2 \quad (17)$$

We have set the ground state to $\varepsilon_0 = 0$ (rather than $\varepsilon_0 = mc^2$), since the absolute energy scale will be irrelevant. Another useful number is the gap to the first excited state

$$\varepsilon_1 = \frac{\hbar^2 \pi^2}{2m L^2} (1, 0, 0)^2 = \frac{\hbar^2 \pi^2}{2m L^2} \quad (18)$$

So that

$$\varepsilon_n = \varepsilon_1 n^2 \quad (19)$$

Then, from Eq. (16) we get

$$N = \sum_{n_x, n_y, n_z=0}^{\infty} \frac{1}{e^{\beta\epsilon_1(n_x^2+n_y^2+n_z^2)} \left(\frac{1}{\langle N_{\text{ground}} \rangle} + 1 \right) - 1} \quad (20)$$

This formula lets us compute N given $\langle N_{\text{ground}} \rangle$ and $\beta\epsilon_1$. For example, if $\langle N_{\text{ground}} \rangle = 80$ and $T = 10 \frac{\epsilon_1}{k_B}$ then doing the sum numerically gives $N = 167.5$. This means with 167.5 particles at $T = 10 \frac{\epsilon_1}{k_B}$, then 80 will be in the ground state.

What we really want is to specify N and T and find $\langle N_{\text{ground}} \rangle$. To get this function, we need to do the sum in Eq. (20) and then solve for $\langle N_{\text{ground}} \rangle$ in terms of $\beta\epsilon_1$ and N . Unfortunately, we cannot do the sum exactly, but at least we can do it numerically. For $N = 100$ we find the numerical solution for $\frac{\langle N_{\text{ground}} \rangle}{N}$ has the form (see the Mathematica notebook on canvas)

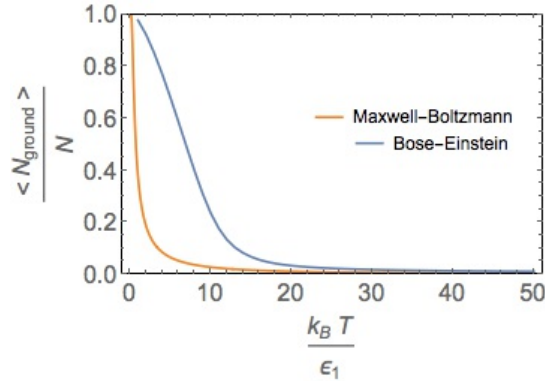


Figure 3. Exact numerical result for the ground state occupancy in a Bose system with $N = 100$.

I added to the plot the prediction using Maxwell-Boltzmann statistics. For MB statistics, we drop all the factors of ± 1 . So, $\langle N_{\text{ground}} \rangle = e^{\beta\mu}$ and so Eq. (20) becomes

$$N = \sum_{n_x, n_y, n_z} \frac{1}{e^{\beta\epsilon_1(n_x^2+n_y^2+n_z^2)} \left(\frac{1}{\langle N_{\text{ground}} \rangle} \right)} = \langle N_{\text{ground}} \rangle \sum_{n_x, n_y, n_z} e^{-\beta\epsilon_1(n_x^2+n_y^2+n_z^2)} \quad (21)$$

These sums can be done numerically with the result is plotted alongside the BE result in Fig. 3. If we turn the sum into integrals, then $\frac{\langle N_{\text{ground}}^{\text{MB}} \rangle}{N} \approx \left(\frac{4\epsilon_1}{\pi k_B T} \right)^{3/2}$ which looks a lot like the exact result that is plotted.

For different N the curve shifts, but looks qualitatively the same. After a little fiddling (inspired by the analytic result below), we see that if we plot the ground state occupancy as a function of $\frac{k_B T}{N^{2/3}\epsilon_1}$ the result is essentially independent of N for the Bose-Einstein case:

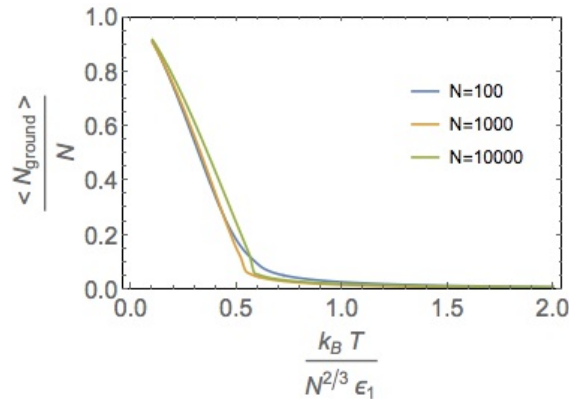


Figure 4. Pulling out a factor of $N^{2/3}$ makes the ground state occupancy essentially independent of N .

3.2 Approximate analytical solution

Having determined the exact solution numerically, let us proceed to use an analytical approach to determine some scaling relations and the transition temperature.

As with the phonon or photon gas, we first transform the sum to an integral via

$$\sum_{\vec{n}} \rightarrow \frac{1}{8} \int_0^\infty 4\pi n^2 dn \quad (22)$$

where the $\frac{1}{8}$ comes from \vec{n} being a vector of whole numbers, as in the phonon or photon case. We want to convert n to ε , which we can do using using Eq. (19), $\varepsilon = \varepsilon_1 n^2$ so

$$d\varepsilon = 2\varepsilon_1 n dn \quad (23)$$

So

$$\sum_{\vec{n}} \rightarrow \frac{\pi}{2} \int_0^\infty n n dn = \frac{\pi}{2} \int_0^\infty \sqrt{\frac{\varepsilon}{\varepsilon_1}} \frac{d\varepsilon}{2\varepsilon_1} = \frac{\pi}{4\varepsilon_1^{3/2}} \int_0^\infty \sqrt{\varepsilon} d\varepsilon \quad (24)$$

As before we write this as

$$\sum_{\vec{n}} \rightarrow \int g(\varepsilon) d\varepsilon \quad (25)$$

where

$$g(\varepsilon) = \frac{\pi}{4\varepsilon_1^{3/2}} \sqrt{\varepsilon} \quad (26)$$

is the density of states.

At this point, we would like to integrate over ε to find N

$$N = \int_0^\infty g(\varepsilon) \langle n_\varepsilon \rangle = \frac{\pi}{4\varepsilon_1^{3/2}} \int_0^\infty d\varepsilon \sqrt{\varepsilon} \frac{1}{e^{\beta(\varepsilon-\mu)} - 1} \quad (27)$$

This is a little too quick, however. The problem is that converting a sum to an integral can only be justified if we do not care at all about the discreteness. For Bose-Einstein condensation we *do* care about the discreteness: the ground state, as we have seen, is qualitatively different from the other states.

Although discreteness is important for the ground states, for the excited states, even the first excited state, there is no issue – the chemical potential μ can never approach any of their energies and so their occupancy numbers will never be unusually large. So let us proceed by taking the continuum limit for all *but* the ground state. Moreover, since $e^{-\beta\mu} \approx 1$ when $\langle N_{\text{ground}} \rangle \gtrsim 1$ which is the region of interest, we can simply set $\mu=0$ for the excited state calculation and Eq. (27) becomes

$$\langle N_{\text{excited}} \rangle = \frac{\pi}{4\varepsilon_1^{3/2}} \int_0^\infty d\varepsilon \sqrt{\varepsilon} \frac{1}{e^{\beta\varepsilon} - 1} = \left(\frac{\pi k_B T}{4\varepsilon_1} \right)^{3/2} \zeta_{3/2} \quad (28)$$

where $\zeta_{3/2} = \zeta\left(\frac{3}{2}\right) \approx 2.61$ with $\zeta(z)$ the Riemann Zeta function.

When does the approximation that $\mu=0$ break down? It breaks down when $\langle N_{\text{ground}} \rangle \approx 0$, as you can see from Eq. (14). As $N_{\text{ground}} \rightarrow 0$ then $\mu \approx -k_B T \ln(N_{\text{ground}})$ which goes to $-\infty$. When $\langle N_{\text{ground}} \rangle \approx 0$ then $\langle N_{\text{excited}} \rangle \approx N$. Setting Eq. (28) equal to N we find that our approximation breaks down when

$$\langle N_{\text{excited}} \rangle = \zeta_{3/2} \left(\frac{\pi k_B T}{4\varepsilon_1} \right)^{3/2} > N \quad (29)$$

We can equivalently write this as $T > T_c$ where the **critical temperature** T_c is defined by

$$N = \zeta_{3/2} \left(\frac{\pi k_B T_c}{4\varepsilon_1} \right)^{3/2} = 2.612 \left(\frac{m k_B T_c}{8\hbar^2 \pi} \right)^{3/2} V \quad (30)$$

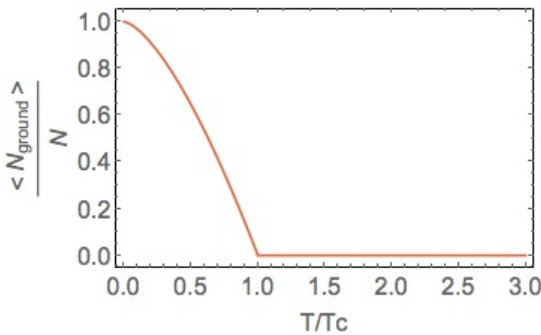
where with ε_1 from Eq. (18) was used. Solving for T_c gives

$$T_c = \frac{4\varepsilon_1}{\pi k_B} \left(\frac{N}{\zeta_{3/2}} \right)^{2/3} = 3.31 \frac{\hbar^2}{k_B m} \left(\frac{N}{V} \right)^{2/3} \quad (31)$$

This lets us write Eq. (28) as

$$\frac{N_{\text{excited}}}{N} = \left(\frac{T}{T_c}\right)^{3/2}, \quad T < T_c \quad (32)$$

Thus, the fraction in the ground state is

$$\frac{\langle N_{\text{ground}} \rangle}{N} = \frac{N - \langle N_{\text{excited}} \rangle}{N} = 1 - \left(\frac{T}{T_c}\right)^{3/2} = \left\langle \frac{N_{\text{ground}}}{N} \right\rangle \quad (33)$$


Comparing to our exact numerical results from Section 3.1 we find good agreement:

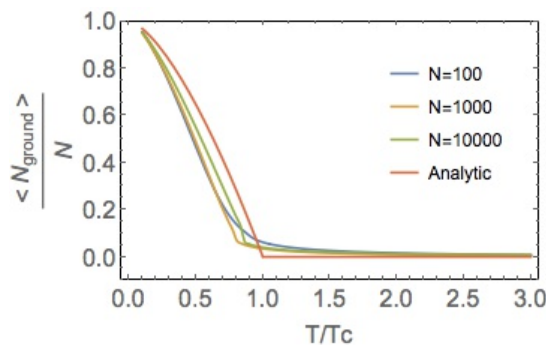


Figure 5. Comparison of the analytic result $1 - \left(\frac{T}{T_c}\right)^{3/2}$ with the exact numerical results for $N = 10^2, 10^3, 10^4$.

4 Experimental evidence

Even though the transition temperature to the BEC is well above the first excited state energy, one still needs to get a system of bosons very very cold to produce a BEC. It took 70 years from when BEC's were first conjectured theoretically (by Satyendra Nath Bose in 1924) to when technology to cool atoms was sufficiently advanced that the BEC could be produced and detected (Cornell, Weimann and Ketterle in 1995).

Since $T_c = 3.31 \frac{\hbar^2}{k_B m} \left(\frac{N}{V}\right)^{2/3}$ having more atoms, a smaller volume, or lighter atoms allows the critical temperature to be higher. You might therefore think hydrogen is the easiest element to cool to see a BEC form. Unfortunately, in the mid 1990s, lasers weren't available that could operate at frequencies conducive to cooling hydrogen. Cornell and Weimann, and Ketterle, used Rubidium atoms, ^{87}Rb to form the BEC. Rubidium has a convenient set of energy levels that were well suited to the available laser cooling technologies at the time. They were able to cool around 2000 atoms using magneto-optical traps (MOTs) along with laser and evaporative cooling techniques to the nanokelvin temperature scales required for the BEC. They found that below $T_c \approx 170\text{nK}$ Bose-Einstein condensation can be seen. This critical temperature is in excellent agreement with the general formula $T_c = 3.31 \frac{\hbar^2}{k_B m} \left(\frac{N}{V}\right)^{2/3}$ when you plug in the density they were able to achieve.

In order to see the BEC, one needs to be able to distinguish atoms in the ground state from atoms in the excited states. The basic feature that makes this possible is that the ground state atoms all have smaller wavenumbers and hence slower velocities than the other states. So if you remove the magnetic trap, the atoms will start to spread, with the ground state atoms spreading more slowly. Thus the atoms' positions after a short time indicate their initial velocities. The scientists photographed (i.e. illuminated the system with a resonant laser pulse) the system after $t = 100\text{ms}$ and found a high density of atoms that had not moved very far:

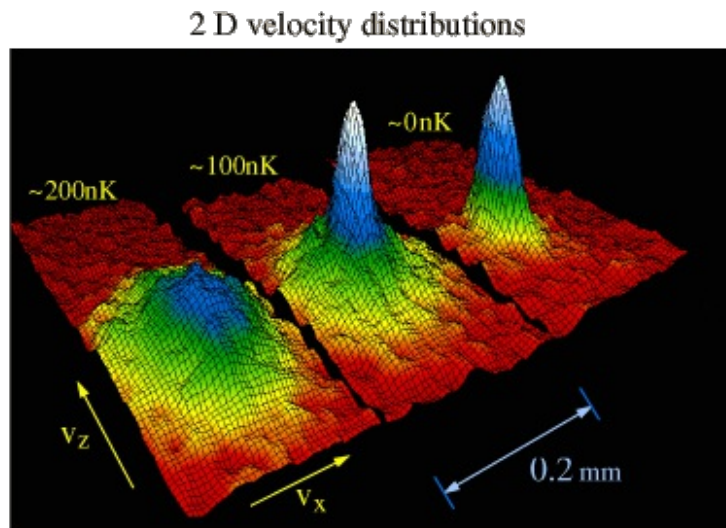


Figure 6. Observation of a Bose-Einstein condensate

In this figure, from Weimann and Cornell's group at the JILA laboratory in Boulder, we see the distribution of atoms in the BEC at different temperatures. The critical temperature is $T_c \approx 170\text{nK}$. Above this temperature (left), the distribution is pretty smooth, consistent with a Maxwell-Boltzmann velocity distribution. Below the critical temperature you can clearly see the higher density corresponding to an anomalously large occupancy of the ground state, consistent with expectations from a BEC. The right image shows the BEC at even lower temperature, where the occupancy is even higher. Direct observation of thermodynamic properties of the system, such as the energy density and heat capacity (particularly by Ketterle's group at MIT), further confirmed that this system was a BEC.

Since their discovery BECs have continued to exhibit some amazing and unusual properties. Because of their coherence (all the atoms are in the *same* state), they can manifest quantum phenomena at larger scales than electrons and are in many ways more controllable than electrons. A number of groups at Harvard and MIT study BECs.

For example, Prof. Greiner (Harvard) uses lasers to localize rubidium atoms in an optical lattice. By controlling the spacing and lattice properties, he can fine tune the system, essentially choosing whatever Hamiltonian he wants. Once the rubidium atoms form a BEC, they exhibit strongly correlated behavior that can be connected to the properties of the Hamiltonian. Such an approach may lead the way to building quantum computers with longer coherence times, or to understanding what material properties might be most likely to produce room-temperature superconductors.

Another example is from Prof. Hau's lab (also at Harvard). In 1999 Prof. Hau constructed a BEC of sodium atoms. Normally, a laser tuned to a hyperfine splitting of the sodium levels would be absorbed and so sodium appears opaque to this frequency. However, Hau was able to entangle the ground and excited states of sodium using photons of a different laser in such a way that she could adjust the transparency of sodium to the first laser. The result is that she could manipulate the dispersion relation of light propagating through the sodium BEC and achieve arbitrarily small group velocities. In her first paper on the subject, she slowed light down to $17\frac{m}{s}$ with this technique. Subsequently, she was able to get light to stop completely.

The BECs produced in laboratories have carefully controlled properties, and very restricted interactions. A BEC of rubidium or sodium is unstable to condense in solid form and be held apart with some careful tricks using magnetic fields and optical traps. Helium is a noble gas that naturally has very weak interactions and will only solidify at very high pressure. At low temperature and pressure, it forms a Bose-Einstein condensate called a **superfluid**. Liquid ^4He is a BEC of helium atoms. Although ^3He is fermionic, pairs of ^3He atoms are bosonic, so liquid ^3He can be thought of as forming due to the pairing of helium atoms. Superfluids have zero viscosity. This is closely related to the bosons all being in the same state, but in this case, the state is not the zero-momentum state but one of non-zero momentum since there is a density current flowing through the fluid.

Bose Einstein condensation is also related to **superconductors**. In a superconductors, like solid mercury at $T < 4.2K$, there is no resistivity. In the BCS theory, the superconductivity is explained through the condensation of pairs of electrons called **cooper pairs**. These pairs act like bosons and form a condensate at low temperature. Because electrons are charged, so are the cooper pairs. The condensation of charged pairs screens the magnetic field in the superconductors, allowing the charged current to flow with zero resistance. Thus superconductors are much like superfluids with charged bosons instead of neutral ones.