NOTE: This is a voluntary exercise! The purpose is to let you get some practise in C++.

1. Level test: Type in the program

```
#include <iostream>
#include <cmath>
using namespace std;
double f(double);
int main()
{
  double x;
  x = 1.12;
  cout<<"f(x) = "<<f(x)<<endl;
}
double f(double x){
  // x^3 +2x^4 + 5x^5
  return pow(x,3)+2.0*pow(x,4)+5.0*pow(x,5);
}
```

compile it, and run it. The result is “f(x) = 13.3637”.
You should understand *everything* in this program before continuing.
Please ask!

CONTINUES ON THE NEXT PAGE

2. In C++ almost everything is a function, so functions take almost anything as arguments. Here we write a function that solves $f(x) = 0$ for *any* continuous function `double f(x)`. For that you need to **pass a function to a function**. Read the lecture notes Chapter 11.4 *Four ways to pass a function to a function.*
I recommend method 3, (*Pass the function as a function object of the class template* ***`std::function`***).
The solver is called either

   - `fail = my::findroot(f, a, b, eps, x)`
     if the derivative of $f(x)$ is not known. If successful, on exit x is the approximate root (use x as a reference).

   - `fail = my::findroot(f,fder,a,b,eps,x)`
     if the derivative of $f(x)$, $f'(x)$, is computed in `fder(x)`. If successful, on exit x is the approximate root.

   Here the return value `fail` is `true` if the routine fails to find a root, and `eps` is the requested accuracy. The name "findroot" is quite common, and chances are it's already used in some library. Put it in your own namespace, such as `my`, to avoid name collision.
   The algorithm could be:

   1. Check that $f(a)f(b) < 0$ to ensure there is at least one root between the limits, $a \leq root \leq b$. This case is handled by setting `fail = true`. One could *throw* an exception. Often such a failure is fatal, so it might be safer stop execution entirely.

   2. If $f'(x)$ is known, try the fast, but potentially unstable, Newton's method (Newton–Raphson), which iterates

      $$x = x - \frac{f(x)}{f'(x)}$$

      The starting point can be $x = \frac{1}{2}(a + b)$. If x is blown away outside the limits $[a, b]$ turn to the mid-point method.

   3. As a fail-safe method, or if $f'(x)$ is not given, use the mid-point method given below.

      $$x = \frac{1}{2}(x_1 + x_2) \ ,$$

      where the root is known to be between x_1 and x_2. if $f(x)f(x_2) < 0$, replace x_1 with x, else replace x_2 with x. Iterate a few times, and try again Newton's method.

   Try, for example, to find a root of $\sin(x)(x^2 + 2x) = 0$.