QM IIa fall 2018, week 4

- Reading assignment for Monday Oct 1st: basics of scattering theory
  - Mathematical methods: contour integration and residue theorem: study from e.g. [Riley, Hobson, Bence, Mathematical methods for physics and engineering, chapter 24, especially 24.12] or [Boas, Mathematical methods in the physical sciences Chapter 14] or lecture notes of Eskola or Tuominen
  - The scattering problem as a time independent Schrödinger equation
  - The Lippmann-Schwinger equation
  - The Born approximation
  - Heikkilä: Chapter II or Tuominen: Secs 4.1-4.5 or Sakurai (old edition): Secs 7.1, 7.2 (see also 7.5) or Sakurai and Napolitano Secs 6.1-6.3 or Eskola, p. 29-73 or Bransden Secs 13.1-13.2 and 13.5-13.6 [also Merzbacher, secs 20.1-20.4].

- Preliminary exercises, do these before the class of Mon Oct 1st and be prepared to present your solutions in class.

  1. Calculate the integral
  $$\int_{-\infty}^{\infty} \mathrm{d}x \frac{e^{ikx}}{x^2+a^2}, \quad k>0, a>0 \tag{1}$$
  using the residue theorem.

  2. Show that the solution to the integral equation [named after whom?]
  $$\psi(\mathbf{r}) = Ce^{i\mathbf{k}\cdot\mathbf{r}} - \frac{1}{4\pi}\int \mathrm{d}^3\mathbf{r}' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} U(\mathbf{r}')\psi(\mathbf{r}') \tag{2}$$
  satisfies the Schrödinger equation
  $$\left[\nabla^2 - U(\vec{r}) + k^2\right]\psi(\vec{r}) = 0 \tag{3}$$
  with the boundary condition $\psi(\vec{r}) = Ce^{i\mathbf{k}\cdot\mathbf{r}}$ for $r \to \infty$. You can *use* the known fact that the function
  $$-\frac{1}{4\pi}\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \tag{4}$$
  satisfies a certain differential equation, which one? We will derive Eq. (4) from this equation in the homework exercises. Does the normalization C matter? Why is there no $\hbar^2/(2m)$ in the kinetic term in Eq. (3)?

  3. From Eq. (2) derive the Born approximation for the *scattering amplitude*. How does one define the *cross section*? How do you get it from the scattering amplitude? What is the energy of the outgoing particle?