QM IIA spring 2020

Exercise 6, discussed in the tutorial session Thu Feb 13th, return by Fri Feb 14th at 21h.

1. Construct the 3-dimensional vectors $|j = 1, m = \pm 1, 0\rangle$ in the usual (x, y, z) basis system. Both $J_\pm|1, m\rangle$ and $J_z|1, m\rangle$ need to give the right thing, according to the Condon-Shortley covention of course! You know that the photon is a $j = 1$ particle. Using these vectors write down radiation gauge vector potentials $\mathbf{A}(\mathbf{x}, t)$ corresponding to the possible helicity states of a photon (helicity is defined as the spin in the direction of motion). In what situation can one have an electromagnetic field that is not a superposition of such photon fields?

2. Construct explicitly the matrices J_i for $j = 3/2$. Check that they obey the commutation relations.

3. (Sakurai, Sec 3, prob. 15). A particle in a spherically symmetric potential $V(r)$ has the wave function

$$\psi(\mathbf{x}) = (x + y + 3z)f(r). \tag{1}$$

   If one measures the angular momentum of this particle, what are the possible values of ℓ and m and their probabilities?

4. An excited one-electron atom is in a $\mathbf{L}^2 = 2(2 + 1)\hbar^2, L_z = 2\hbar$ state. The angular momentum is measured with respect to an axis pointing in the direction $\mathbf{n} = (\sin\beta, 0, \cos\beta)$. What are the probabilities for obtaining the different possible values of $\mathbf{n} \cdot \mathbf{L}$?

5. Show that the matrix that rotates 3-dimensional vectors by and angle ω around the axis $\mathbf{n}$ is

$$R(\mathbf{n}, \omega)_{ij} = \cos\omega\delta_{ij} + (1 - \cos\omega)n_i n_j - \sin\omega\varepsilon_{ijk}n_k. \tag{2}$$

   The generators here are $(J_i)_{kl} = -i\varepsilon_{ijk}$. Using this result and the vectors from problem 1 calculate $d^1_{m',m}(\theta)$.