QM II spring 2020

Exercise 3, discussed in the tutorial session Thu Jan 23rd, return by Fri Jan 24th by 21h and again self-graded on Mon Jan 27th by 14h.

1. Consider charged particles (satisfying the Pauli equation) in a magnetic field.
   (a) Show that the gauge potential $\mathbf{A} = (0, A^y = xB, 0)$ corresponds to a magnetic field of magitude B in the z-direction. You know that the classical trajectory of a particle with mass m, electric charge e and velocity $\mathbf{v}$ with $v_z = 0$ is a circular motion, what is its angular frequency ω_B?
   (b) Using the ansatz $\psi(\mathbf{x}) = e^{i(k_z z + k_y y)}\phi(x)$ in the time-independent Pauli equation in this magnetic field show that you get a harmonic oscillator equation of motion. Where in x is the minimum of the harmonic potential? What is the difference between two energy levels in terms of ω_B? The energy levels are known as *Landau levels*. You can neglect the spin.
   (c) What kind of an equation would you get with $\mathbf{A} = (A^x = -yB/2, A^y = xB/2, 0)$? You do not need to solve it, but discuss how it should give the same energy level splitting.

2. Assume that a hydrogen atom is in the state $|n\ell m\rangle = |100\rangle$ at $t = -\infty$. It is subjected to an electric field pulse in the z-direction with

$$E_z(t) = E_0 e^{-t^2/\tau^2} \tag{1}$$

   (a) Calculate the transition probability to the state $|n\ell m\rangle$ at $t = \infty$ in a gauge where $\varphi \neq 0, \mathbf{A} = \mathbf{0}$, at lowest order in perturbation theory. Express the result in terms of the dipole matrix element $\langle n\ell m|z|100\rangle$ (which you do not need to calculate).
   (b) Show, using the explicit forms of the angular parts of the wavefunctions, that the only $n = 2$ state to which there is a nonzero transition probability is $\ell = 1, m = 0$ because of the angular integrals.

   Bonus points: evaluate also the radial integral to obtain the value $\langle 210|z|100\rangle$; see see e.g. the Griffiths book for the explicit form of the wavefunctions.

3. Consider again the electric field pulse of problem 2. Calculate the transition probability at $t = \infty$ in a gauge where $\varphi = 0, \mathbf{A} \neq \mathbf{0}$, at lowest order in perturbation theory, in terms of the same matrix element $\langle 210|z|100\rangle$. You might have to multiply $\mathbf{A}$ by $e^{-\varepsilon t}$ to make the integrals converge, but check that you can take $\varepsilon \to 0$ in the end; why is this?

4. A hydrogen atom in its ground state is put in long wavelength (compared to the Bohr radius a_0) electromagnetic radiation with a frequency ω and an intensity I. Calculate its photoelectric ionization rate. Compare the angular distribution of the outgoing electron to the result of last week's homework exercise in the corresponding $k = 0$ limit. See lecture notes of Tuominen.

5. An infinite well potential expands adiabatically from width w_1 to width w_2.
   (a) Calculate the geometric phase for an eigenstate under this expansion
   (b) If the expansion occurs at a constant rate $\mathrm{d}w/\mathrm{d}t = v$ =cst, what is the *dynamic* phase change?

   (Griffiths, exercise 10.3)