QM IIA spring 2020, week 6

Reading assignment for Tuesday Feb 11th: rotations

- Groups and representations
- Rotation matrices
- Angular momentum algebra (reminder from QMI)
- Read from:
  - Tuominen: Secs 5.1-5.4
  - Sakurai (old edition or Sakurai & Napolitano): Secs 3.1-3.3, 3.5-3.6
  - Bransden Secs 6.1-6.9
  - Eskola, p. 241-285
  - Niskanen Secs. 2.1-2.5

Preliminary exercises do these before the class of Tue Feb 11th and be prepared to present your solutions in class.

1. (a) What is a group? What is a representation? Why do the matrices that rotate 3-dimensional vectors form a group? What is the name of this group?

   (b) What are the matrices for infinitesimally small rotations of a 3-dimensional vector around the x, y and z axes? From these identify the generators of the group. What are the commutation relations of the generators?

2. Recall the spectrum of the angular momentum operator, taking $\hbar = 1$.

   (a) Why do the operators $\mathbf{J}^2$ and J_z have simultaneous eigenstates: $|\lambda_{\mathbf{J^2}}, \lambda_{J_z}\rangle$ with $\mathbf{J}^2|\lambda_{\mathbf{J^2}}, \lambda_{J_z}\rangle = \lambda_{\mathbf{J^2}}|\lambda_{\mathbf{J^2}}, \lambda_{J_z}\rangle$ and $J_z|\lambda_{\mathbf{J^2}}, \lambda_{J_z}\rangle = \lambda_{J_z}|\lambda_{\mathbf{J^2}}, \lambda_{J_z}\rangle$?

   (b) What are the possible eigenvalues $\lambda_{\mathbf{J^2}}$ and λ_{J_z}, how does one find them from the commutation relations?

3. What is the matrix that rotates a $j = 1/2$ spinor by an angle ω around the axis $\mathbf{n}, \mathbf{n}^2 = 1$? What happens with $\omega = 2\pi$? What is the matrix that rotates the same spinor by first an angle γ around the $z-$axis, then an angle β around the y-axis and finally an angle α around the z-axis? Convince yourself that all rotations can be expressed in both ways. What is $d^j_{m,m'}(\beta)$? Read off from your result $d^{1/2}_{1/2,1/2}$ and $d^{1/2}_{1/2,-1/2}$.

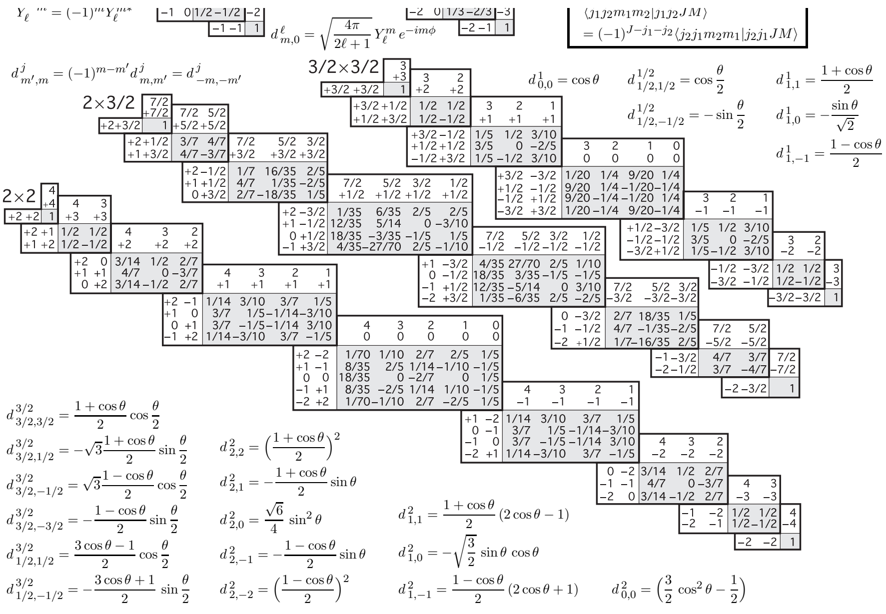

Figure 44.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).