QM IIA spring 2020, week 4

Reading assignment for Tuesday Jan 28th: basics of scattering theory

- Mathematical methods: contour integration and residue theorem. If you are not already familiar with the basics, study from e.g. [Riley, Hobson, Bence, Mathematical methods for physics and engineering, chapter 24, especially 24.12] or [Boas, Mathematical methods in the physical sciences Chapter 14] or lecture notes of Tuominen or Eskola
- The scattering problem as a time independent Schrödinger equation
- The Lippmann-Schwinger equation
- The Born approximation
- Read from
  - Heikkilä Chapter II
  - Tuominen: Secs 4.1-4.5
  - Sakurai (old edition): Secs 7.1, 7.2 (see also 7.5)
  - Sakurai and Napolitano Secs 6.1-6.3
  - Bransden Secs 13.1-13.2 and 13.5-13.6
  - Eskola, p. 29-73

Preliminary exercises Do these before the class of Tuesday Jan 28th and be prepared to present your solutions in class.

1. Calculate the integral
$$\int_{-\infty}^{\infty} \mathrm{d}x \frac{e^{ikx}}{x^2+a^2}, \quad k>0, a>0 \tag{1}$$
using the residue theorem.

2. Show that the solution to the integral equation [named after whom?]
$$\psi(\mathbf{r}) = Ce^{i\mathbf{k}\cdot\mathbf{r}} - \frac{1}{4\pi}\int \mathrm{d}^3\mathbf{r}' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} U(\mathbf{r}')\psi(\mathbf{r}') \tag{2}$$
satisfies the Schrödinger equation
$$\left[\nabla^2 - U(\vec{r}) + k^2\right]\psi(\vec{r}) = 0 \tag{3}$$
with the boundary condition $\psi(\vec{r}) = Ce^{i\mathbf{k}\cdot\mathbf{r}}$ for $r\to\infty$. You can *use* the known fact that the function
$$-\frac{1}{4\pi}\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \tag{4}$$
satisfies a certain differential equation, which one? We will derive Eq. (4) from this equation in the homework exercises. Does the normalization C matter? Why is there no $\hbar^2/(2m)$ in the kinetic term in Eq. (3)?

3. From Eq. (2) derive the *Born approximation* for the *scattering amplitude*. How does one define the *cross section*? How do you get it from the scattering amplitude? What is the energy of the outgoing particle?