QM IIa spring 2020, week 3

Reading assignment for Tuesday Jan 21st: coupling of matter and classic electromagnetic field

- Minimal substitution, Pauli equation
- Gauge invariance, Berry's phase, Aharonov-Bohm effect, flux quantization
- Radiation field, dipole approximation
- Read from at least one of
  - Tuominen: Secs 3.5-3.8
  - Heikkilä: Secs III.5-III.6
  - Sakurai: Secs 2.6, 5.7 and Supplement I (old edition), 2.7, 5.6 and 5.8 (new edition)
  - Bransden Chap 11 & 12
  - Eskola, p. 149-213
  - Niskanen, secs 8.1, 8.3, 8.6.5
  - For Berry's phase see also Griffiths, Chapter 10

Preliminary exercises Do these before the class of Tuesday Jan 21st and be prepared to present your solutions in class.

1. Show that the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ are invariant in a gauge transformation. Then show that the Pauli equation is invariant under a gauge transformation.
2. Construct a *radiation gauge* vector potential $\varphi(t, \mathbf{x}), \mathbf{A}(t, \mathbf{x})$ describing monochromatic light with a wavelength $\lambda = 400\text{nm}$ (this is visible light) and intensity 100W/m^2 propagating in the z-direction. You might need to recall the intensity of electromagnetic radiation, possibly from the materials for an electrodynamics course. Beware of different electromagnetic unit systems in different sources (e.g. Sakurai looks different, see discussion in Appendix of Niskanen)! You can also give a numerical value, but more important than this is that you understand whatever unit system you are working in. Is the vector potential "small" or "large" when one studies the interaction of this radiation with ordinary matter? What should one compare it to?
3. Show that the transition rates between states of an atom interacting with electromagnetic radiation such as the one in the previous problem can be expressed in terms of matrix elements of the dipole operator.