QM IIa spring 2020, week 2 [typos corrected 14.1.2020]

Reading assignment for Tue Jan 14th, for class at 12h:

- Schödinger, Heisenberg and interaction pictures
- Time-independent perturbation: Fermi's golden rule
- Harmonic time-dependent perturbation (needed for written exercises)
- Read from at least one of
  - Heikkilä: Secs III.1-III.4
  - Tuominen: Secs 3.1-3.4
  - Sakurai: Secs 2.1-2.4 + 5.5, 5.6 and 5.8 (old edition), (+ 5.5, 5.7 and 5.9 new edition)
  - Bransden Chap 9
  - Eskola, p. 100-148
  - Niskanen, secs 8.4, 8.5

Preliminary exercises After reading the assignment work these out before the class of Tue Jan 14th and be prepared to present your solutions in class.

1. Consider a spin 1/2 particle in an external magnetic field in the z-direction. Simplify a bit the constants, setting $\hbar = 1$ and assuming that the system is described by a Hamiltonian $\hat{H} = \sigma_z$, with σ_z the Pauli matrix. Originally $t = 0$ the system is in a state with positive spin in the x-direction
$$\frac{1}{\sqrt{2}} \left(|\uparrow\rangle + |\downarrow\rangle \right). \tag{1}$$
   (a) Solve the Schrödinger equation of motion and calculate $\langle \sigma_x \rangle$ as a function of t.
   (b) Calculate $\langle \sigma_x(t) \rangle$ in the Heisenberg picture.

2. Solve the differential equation
$$\frac{\mathrm{d}}{\mathrm{d}t} U(t) = -iA(t)U(t), \tag{2}$$
with the initial condition $U(t = 0) = 1$, when $A(t)$ some known time-dependent matrix. What is U? What is A? First write the solution in series form, who is this series named after? Then rewrite it as a time ordered exponential.

3. Now specialize to the case ($\hbar = 1$)
$$A(t) = \overbrace{\exp\left\{ i \begin{pmatrix} \epsilon_i & 0 \\ 0 & \epsilon_f \end{pmatrix} t \right\}}^{\equiv U_0^\dagger} \overbrace{\begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix}}^{V_S} \overbrace{\exp\left\{ -i \begin{pmatrix} \epsilon_i & 0 \\ 0 & \epsilon_f \end{pmatrix} t \right\}}^{\equiv U_0} \theta(t) \tag{3}$$
with the initial condition $U(t = 0) = 1$ and calculate, for $t > 0$ and *to lowest order in a i.e. lowest order in A*
$$c(t) \equiv (0 \quad 1) U(t) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{4}$$
and
$$\lim_{t \to \infty} \frac{|c(t)|^2}{t} \tag{5}$$
using
$$\lim_{t \to \infty} \frac{\sin^2 tE}{E^2 t} = \pi \delta(E) \tag{6}$$
What is the interpretation of U_0, V_S ? What is the quantity (4)? With what name is the result (5) known? Interpret as a time-energy uncertainty relation. What happens when the perturbation has a harmonic time dependence
$$V_S = \begin{pmatrix} 0 & ae^{i\omega t} \\ ae^{-i\omega t} & 0 \end{pmatrix} \quad ? \tag{7}$$