Far-from-equilibrium universality classes
In heavy-ion collisions and cosmology

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J. Berges, KB, S. Schlichting and R. Venugopalan, *PRD 92, 096006 (2015); PRL 114, 061601 (2015)*

A. Piñeiro Orioli, KB and J. Berges, *PRD 92, 025041 (2015)*

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Raju Venugopalan
Roland Walz

Particle Physics Day

November 25, 2016

An overview:

KB, *PhD thesis (2016)*
(linked e.g. via HEP-INSPIRE)

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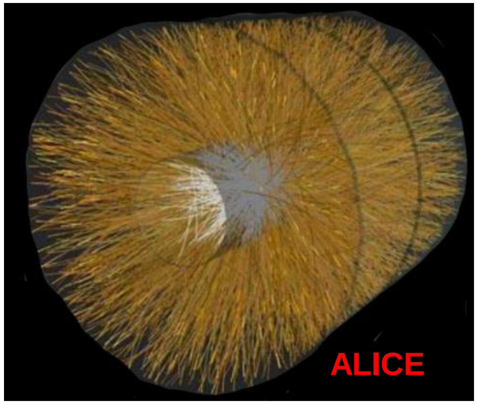
Introduction: Motivation

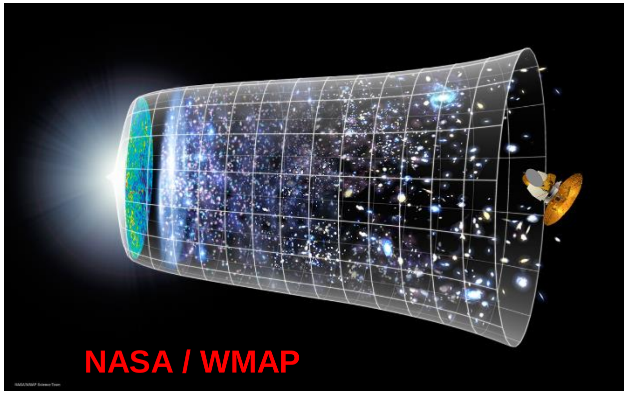
Systems far from equilibrium

Heavy-ion collisions



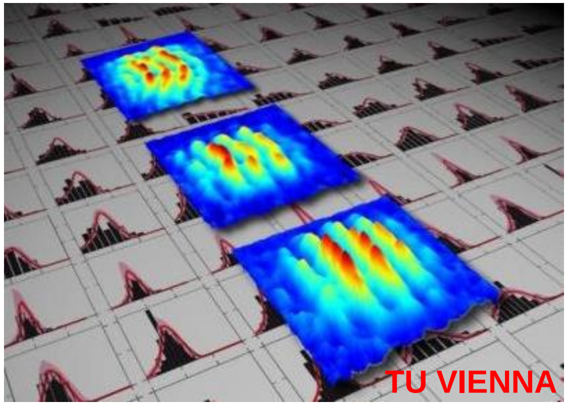
(Expanding) non-Abelian gauge theory

Inflationary cosmology



Relativistic O(N) scalars

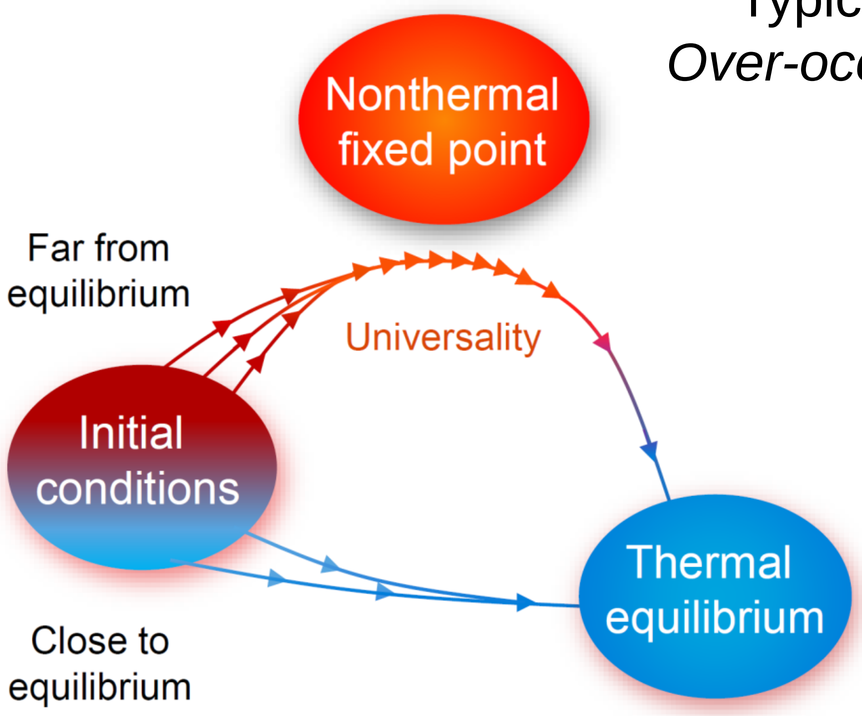
Ultracold atoms



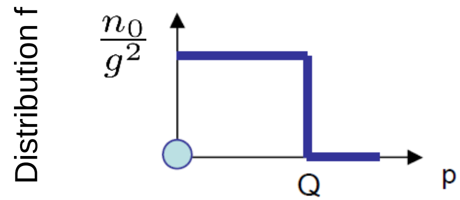
Nonrelativistic scalars

For weak couplings: Universal properties?

Introduction: Nonthermal fixed points



Typical IC:
Over-occupation



$$f(p, t_0) = \frac{n_0}{\lambda}\Theta\left(Q - p\right)$$

Nonthermal fixed point (NTFP)

Micha & Tkachev, *PRD* 70, 043538 (2004)

Berges, Rothkopf & Schmidt, *PRL 101, 041603 (2008)*

- ✓ Partial memory loss
- ✓ Time scale independence
- ✓ Self-similar dynamics

(Single-particle) Distribution function: $f(p,t) = t^{\alpha} f_S\left(t^{\beta} p\right)$

Introduction: Simulation method

Weak couplings $g^2, \lambda \ll 1$ but large occupancies $f \gg 1$

Classical-statistical simulations

- Initialize fields $\varphi(\boldsymbol{x}, t_0)$ or $A(\boldsymbol{x}, t_0)$
- Solve classical equations of motion on the lattice
- Observables averaged over (quantum) IC

Classicality condition:

$$f(p,t) = \sqrt{\langle \varphi\varphi \rangle \langle \partial_t \varphi \, \partial_t \varphi \rangle} \gg 1$$

Aarts & Berges,
PRL 88, 041603 (2002)

Examples: *Micha & Tkachev ; Smit & Tranberg; Nowak, Scholle, Sexty & Gasenzer ; Berges, KB, Schlichting & Venugopalan; Kurkela & Moore; …*

Universality class: scalars in the IR

Massless relativistic scalars

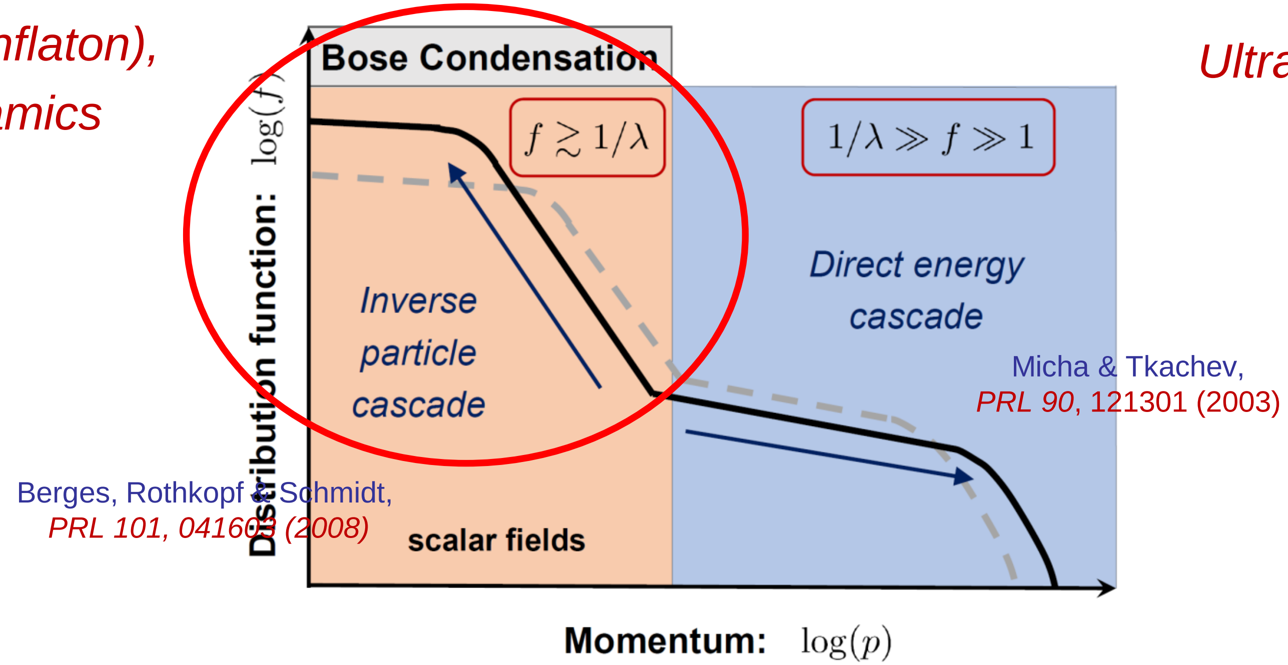
O(N) – symmetric, $\lambda\varphi^4$ interaction

Cosmology (inflaton), Higgs dynamics

Nonrelativistic scalars

(Gross-Pitaevskii)

Ultracold atoms



A. Piñeiro Orioli, KB, and J. Berges,
PRD 92, 025041 (2015)

Universality class: scalars in the IR

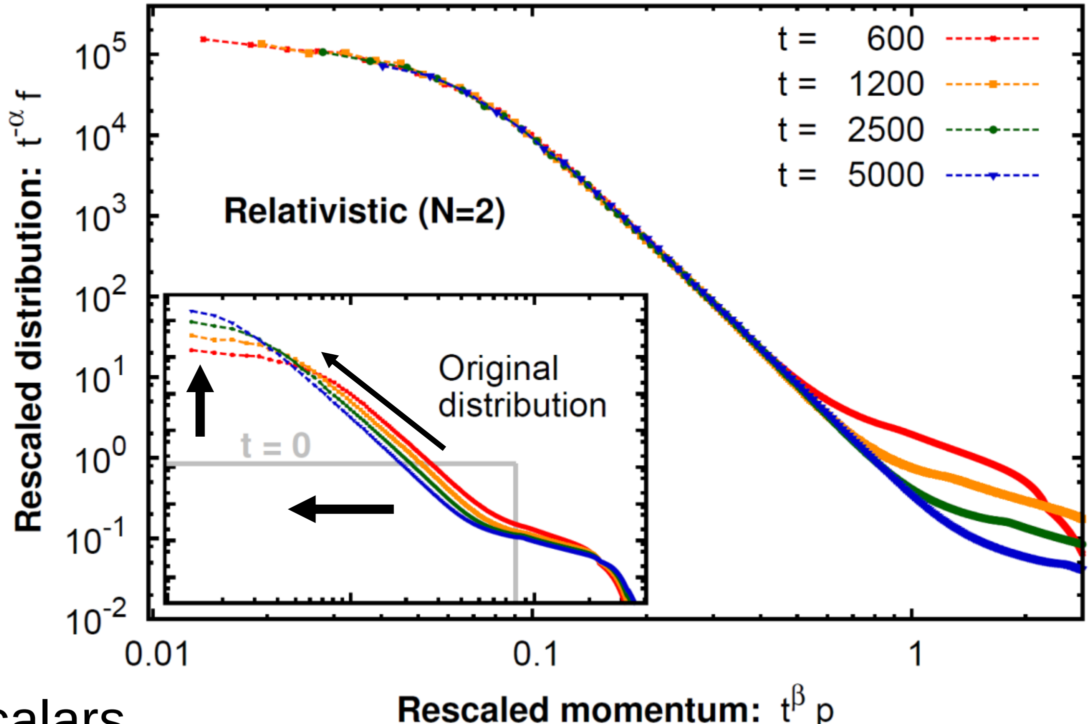
Pinerio Orioli, KB & Berges,
PRD 92, 025041 (2015)

Self-similar evolution

$$f(p,t) = t^{\alpha} f_S\left(t^{\beta} p\right)$$

Self-similar evolution

Inverse particle cascade to infrared



Scaling exponents

$$\boxed{\alpha = 3/2\,,\ \beta = 1/2}$$

Particle number conservation

$$n \sim \int d^3p f = const$$

For nonrelativistic and relativistic O(N) symmetric scalars

Universality class: scalars in the IR

Self-similar evolution

$$f(p,t) = t^{\alpha} f_S\left(t^{\beta} p\right)$$

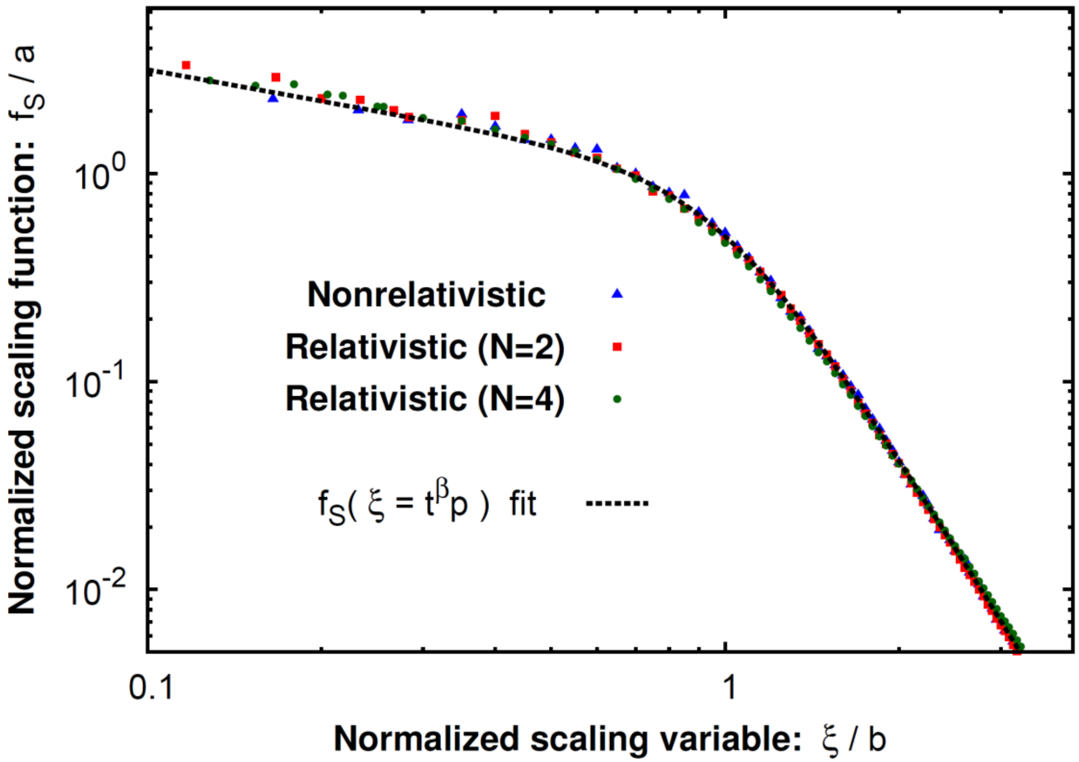
Universal scaling function

Pinerio Orioli, KB & Berges,
PRD 92, 025041 (2015)

Same functional form

$$\lambda f_S = \frac{a}{(|\mathbf{p}|/b)^{\kappa_<} + (|\mathbf{p}|/b)^{\kappa_>}}$$

with $\kappa_< \simeq 0.5\,,\ \kappa_> \simeq 4.5$



Universality class for:

- non-relativistic Bose gases
- O(N) symmetric scalars

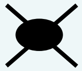
Universality class: scalars in the IR

Explanation: Vertex-resummed kinetic theory

Kinetic theory for elastic collisions

$$\boxed{\frac{\partial f(\mathbf{p},t)}{\partial t} = C^{2\leftrightarrow 2}[f;\lambda](\mathbf{p},t)}$$

At low momenta high occupancies → resummation needed
(because of interference of infinitely many scattering processes)

$\lambda^2 \mapsto \lambda^2_{\mathrm{eff}}[f](p) \sim \Bigg|$  $\Bigg|^2 = \dfrac{\lambda^2}{|1+\lambda\Pi^R(p)|^2}$

Berges, Rothkopf & Schmidt,
PRL 101, 041603 (2008)

Berges & Sexty,
PRD 83, 085004 (2011)

Leads to correct scaling exponents!

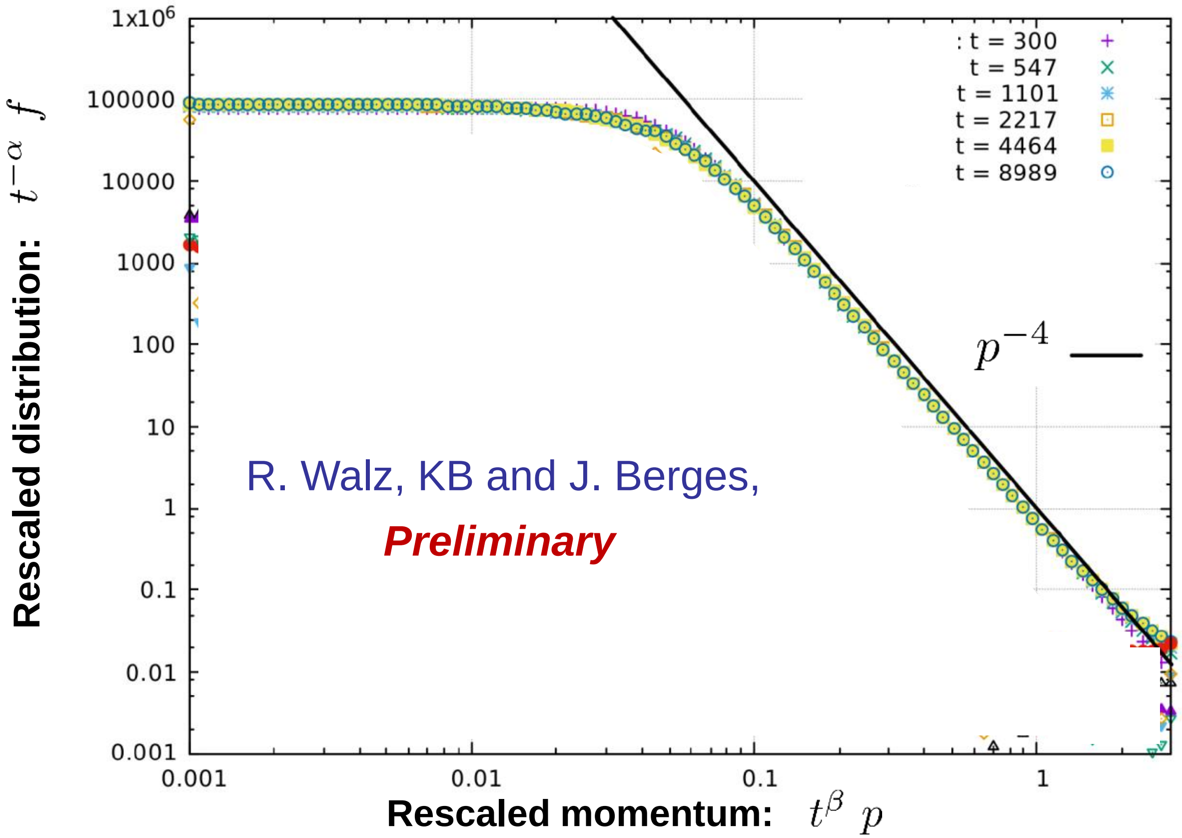
$$\boxed{\alpha = 3/2\,,\ \ \beta = 1/2}$$

Pinerio Orioli, KB & Berges,
PRD 92, 025041 (2015)

Universality class: scalars in the IR

Explanation: Vertex-resummed kinetic theory

Nonrelativistic scalars in VR kinetic theory



Self-similar evolution

$$f(p,t) = t^{\alpha} f_S\left(t^{\beta} p\right)$$

Effective kinetic theory also captures scaling form!

$$f_S \approx \frac{a}{(|\mathbf{p}|/b)^{\kappa_<} + (|\mathbf{p}|/b)^{\kappa_>}}$$

with $\kappa_< \approx 0\,, \quad \kappa_> \approx 4$

Universality class: expanding gauge & scalar theories

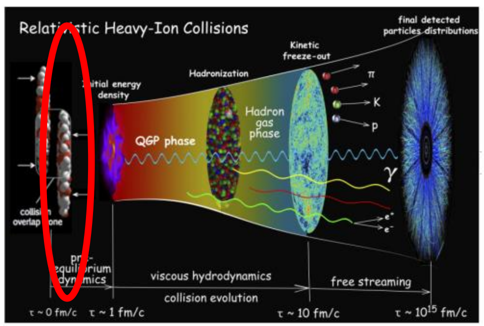
Massless scalar field theory (O(N))

$$S = \int d\tau d^2 x_T d\eta \, \tau \left(\frac{g^{\mu\nu}}{2} (\partial_\mu \varphi_a)(\partial_\nu \varphi_a) - \frac{\lambda}{4!N} (\varphi_a \varphi_a)^2 \right)$$

Non-Abelian gauge theory (SU(2))

$$S = \int d\tau d^2 x_T d\eta \, \tau \, F^a_{\mu\nu} F^{a,\mu\nu}$$

Heavy-ion collisions at early times (weak-coupling limit $\alpha_s \ll 1$)



Little Bang by P. Sorensen and C. Shen

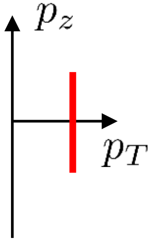
Bjorken coordinates: $\tau = \sqrt{t^2 - (x^3)^2}\,, \quad \eta = \operatorname{artanh}\left(\frac{x^3}{t}\right)$

Metric in Bjorken coordinates:
longitudinal expansion (in beam direction) $g_{\mu\nu}(\tau) = \operatorname{diag}\,(1, -1, -1, -\tau^2)$

Self-similar evolution: $f(p_T, p_z, \tau) = \tau^\alpha f_S\left(\tau^\beta p_T, \tau^\gamma p_z\right)$

J. Berges, KB, S. Schlichting and R. Venugopalan:
PRL 114, 061601 (2015) ;
PRD 92, 096006 (2015)

Universality class: expanding gauge & scalar theories



Longitudinal dynamics

Reminder: *Self-similar evolution*

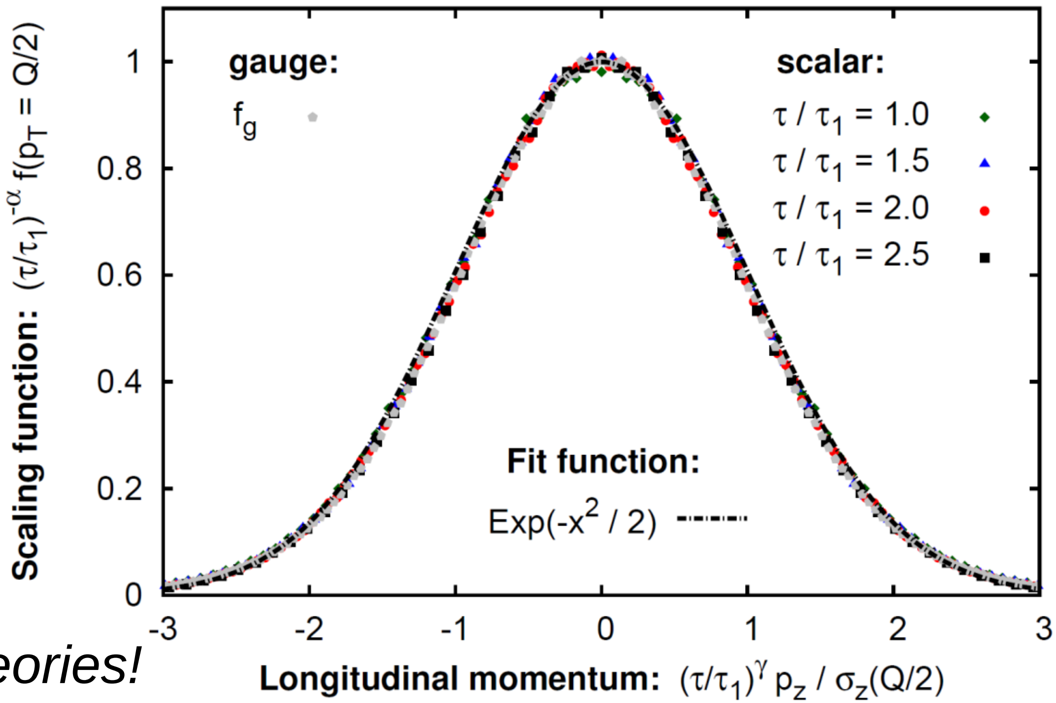
$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

Shared universality class

Dynamical exponents

$$\boxed{\begin{aligned} \alpha &= -2/3 \\ \beta &= 0 \\ \gamma &= 1/3 \end{aligned}}$$

Gaussian scaling form



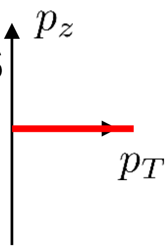
Same exponents and scaling function in gauge and scalar theories!

Gauge attractor from elastic scatterings

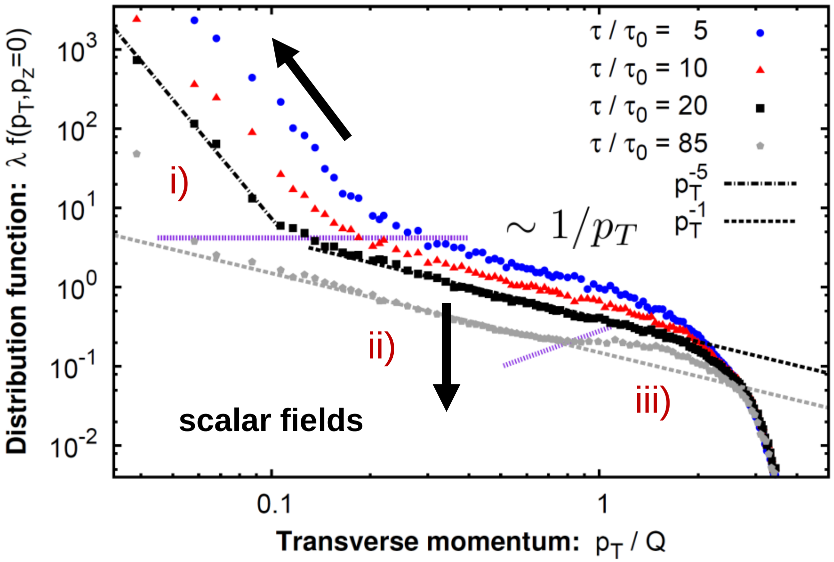
Baier, Mueller, Schiff & Son, PLB 502, 51 (2001)

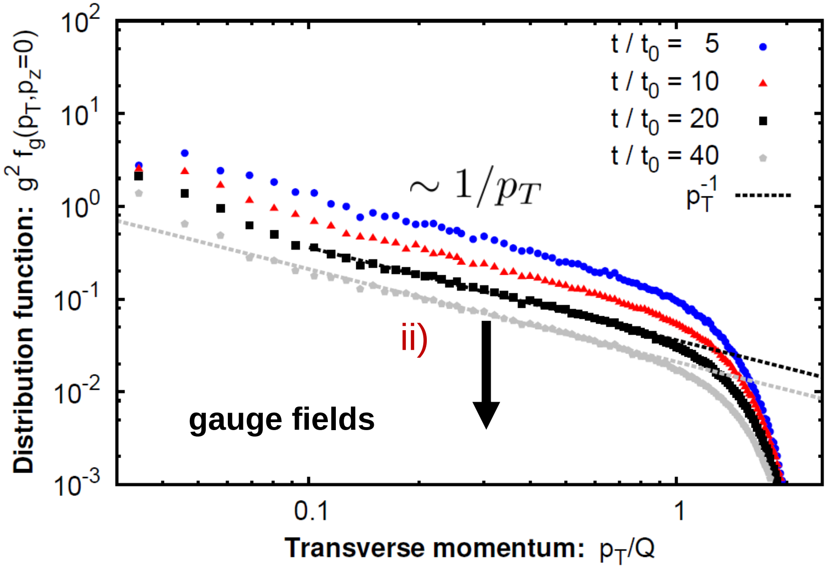
Berges, KB, Schlichting, Venugopalan, *PRD 89, 074011 + 114007 (2014)*

Universality class: expanding gauge & scalar theories



Where is shared scaling region?





- Scaling range ii) is the universal region.

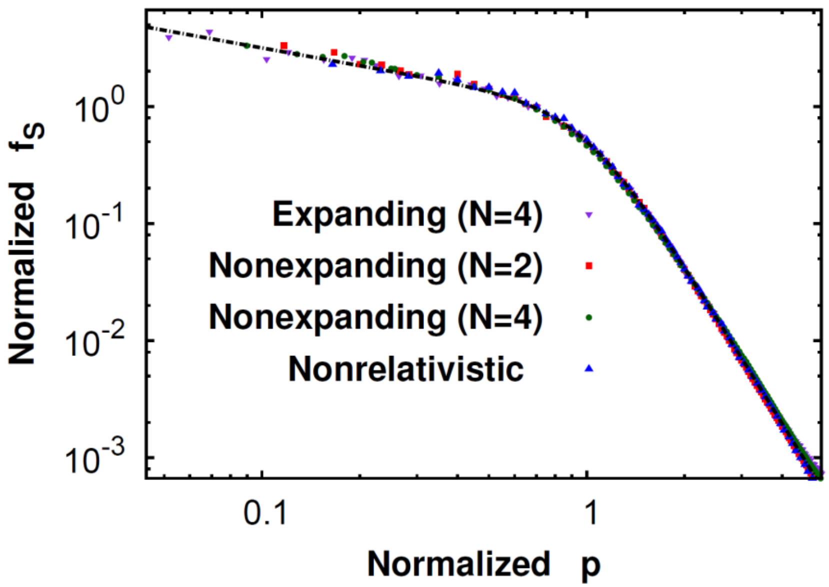
- Exponents and form largely insensitive to initial conditions (memory loss), see

J. Berges, KB, S. Schlichting, and R. Venugopalan: *PRD 89, 074011 + 114007 (2014) ; PRD 92, 096006 (2015)*

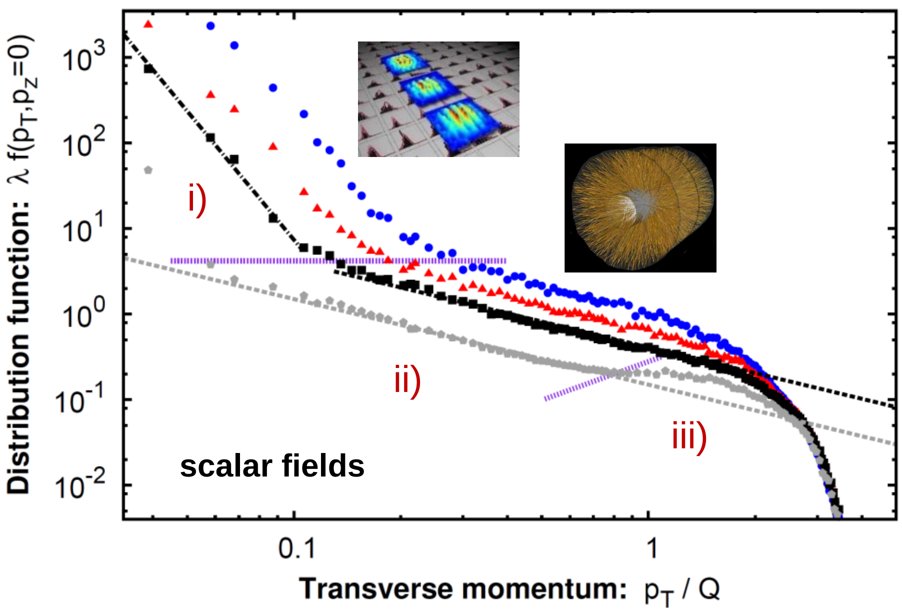
Expanding scalars – linking universal features

Link between the world‘s hottest and coldest matter

Scalar fields infrared scaling region: i)



Longitudinally expanding scalars



- ✓ Broad universality class
- ✓ *Explanation:* vertex-resummed kinetic theory

i. Cold atoms, inflationary cosmology
ii. Heavy-ion collisions

Berges, KB, Schlichting & Venugopalan, *PRD 92, 096006 (2015)*

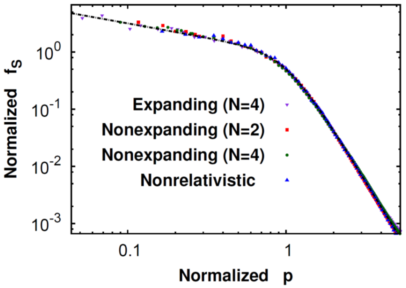
See also: KB, *PhD thesis (2016)*

Conclusion

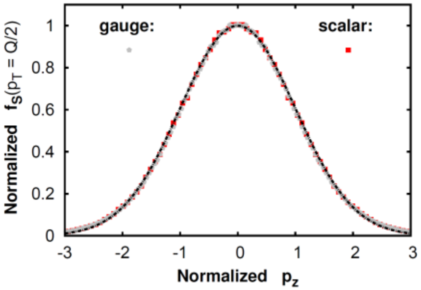
- Over-occupied quantum systems may approach nonthermal fixed-points
- Scalar systems lie in the same infrared universality class
- Expanding scalar and gauge systems share a universality class

Outlook

- Possibility of accessing infrared in inflationary cosmology with ultracold atoms
- Simulation of inflaton evolution with resummed kinetic theory?
- How can far-from-equilibrium universality classes be classified?
- Our results hint at a nontrivial infrared region in gauge theories → description?

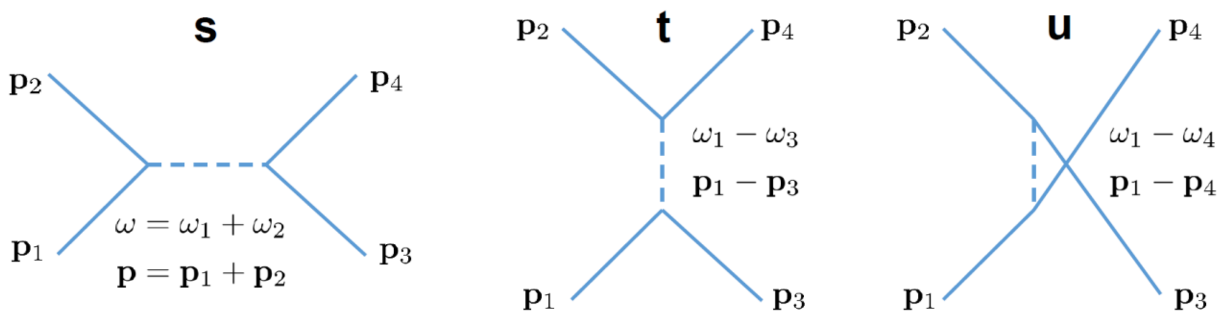


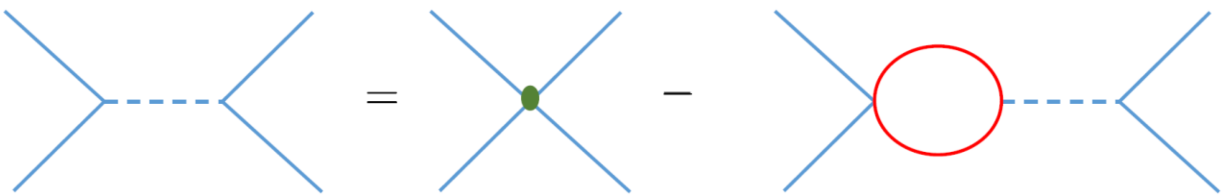
Thank you for your attention!



BACKUP SLIDES

Details on vertex-resummation



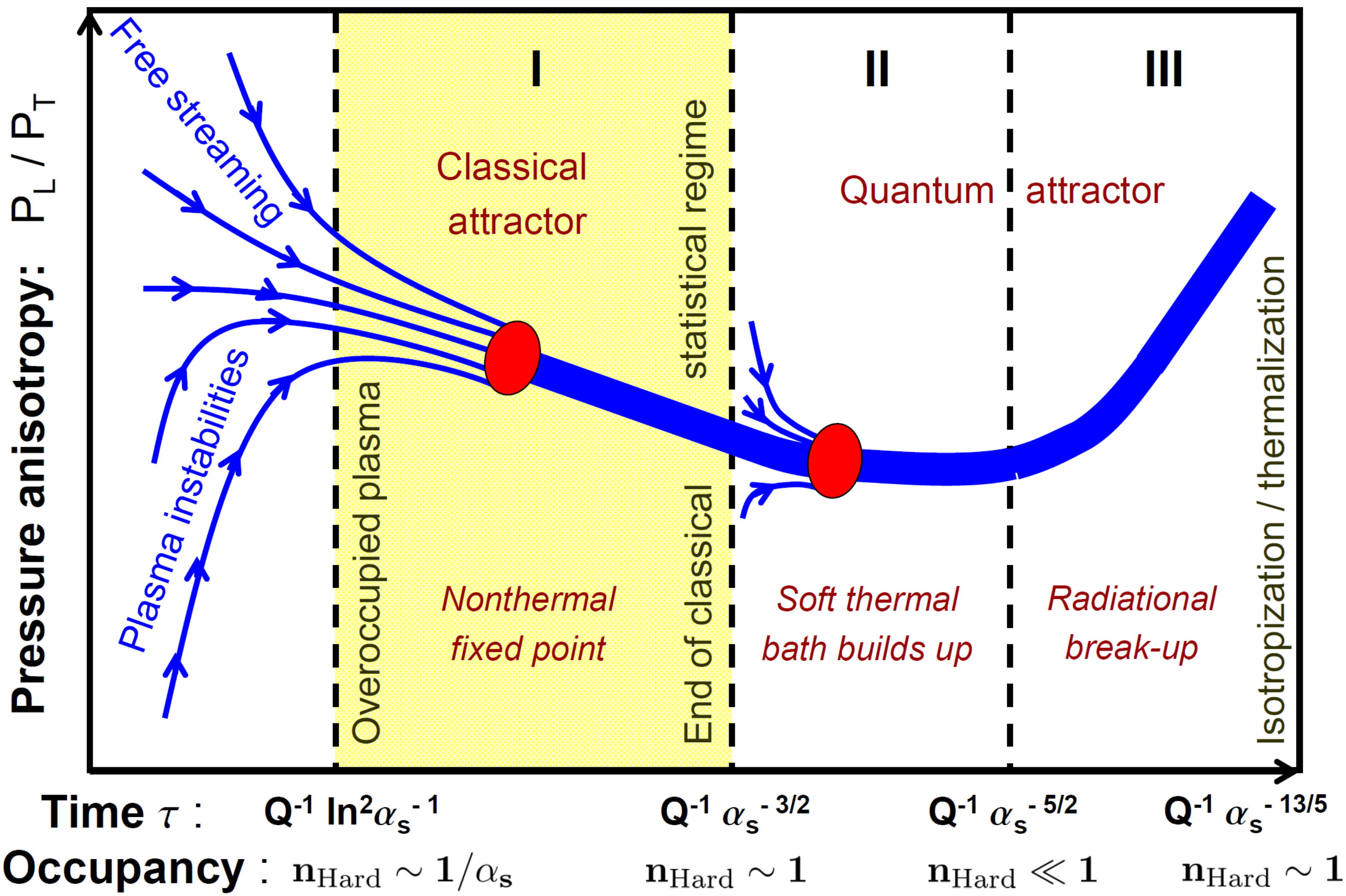


$$\lambda_R[f](\omega, \mathbf{p}, t) \quad = \quad \lambda \quad - \quad \Pi^R[f](\omega, \mathbf{p}, t) \;\; \lambda_R[f](\omega, \mathbf{p}, t)$$

$$|\lambda_{\text{eff}}|^2 = \frac{1}{3}\left(|\lambda_{\text{R}}(s)|^2 + |\lambda_{\text{R}}(t)|^2 + |\lambda_{\text{R}}(u)|^2\right)$$

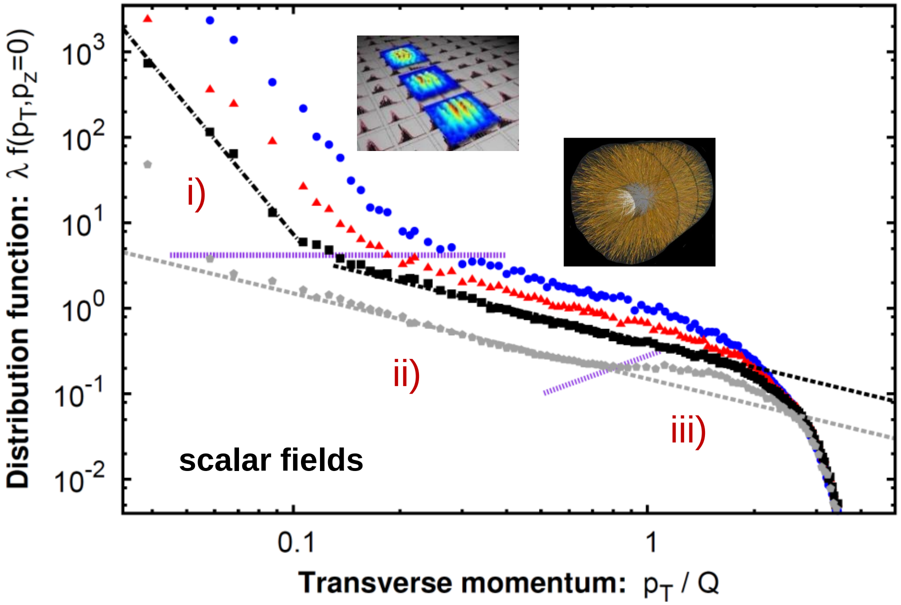
Thermalization scenario of non-Abelian plasmas

(a la `bottom-up')



Expanding scalars – linking universal features

The entire attractor in longitudinally expanding scalars



Reminder: *Self-similar evolution*

$$f(p_T, p_z, \tau) = \tau^\alpha f_S(\tau^\beta p_T, \tau^\gamma p_z)$$

Berges, KB, Schlichting & Venugopalan,
PRD 92, 096006 (2015)

Scaling regions	α	β	γ	λf_S
i)	1	$2/3$	$2/3$	$\left((p/b)^{-1/2} + (p/b)^{-5}\right)^{-1}$
ii)	$-2/3$	0	$1/3$	$p_T^{-1}\ e^{-p_z^2/2\sigma_z^2}$
iii)	$-1/2$	0	$1/2$	$\mathrm{sech}(p_z/\sigma_z)$

Expanding scalars – linking universal features

Puzzles

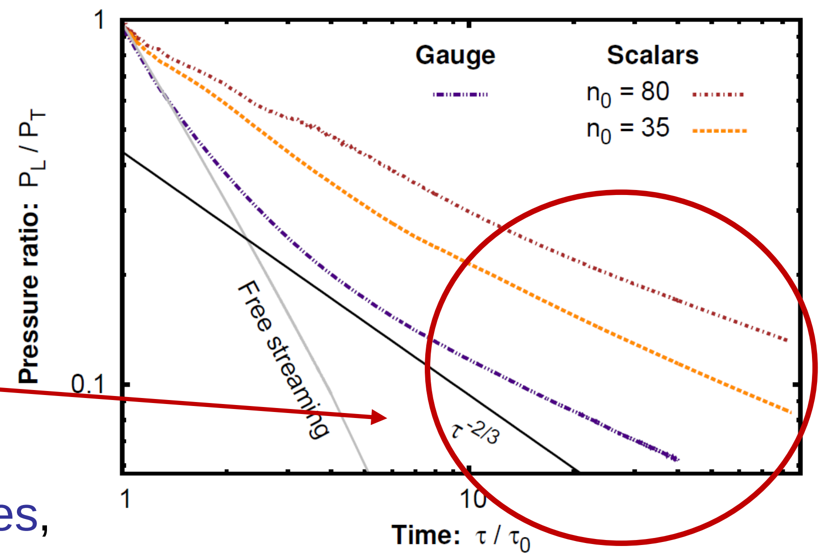
Berges, KB, Schlichting, Venugopalan,
PRD 92, 096006 (2015)

Pressure ratio:

Parametrically:

$$\frac{P_L}{P_T} \overset{\text{kinetic theory}}{\sim} \frac{\int d^3p\, f\, p_z^2/\omega}{\int d^3p\, f\, p_T^2/\omega} \overset{\text{late times}}{\sim} (Q\tau)^{-2/3}$$

Visible discrepancies!



P_L: important contributions from IR modes,
Described by vertex-resummed kinetic theory (scalars)

- Results challenge kinetic description of non-Abelian gauge theory.
- IR in non-Abelian plasmas isotropic? Enhanced? Transient condensation? Nontrivial topological structure? Vertex-resummations required?