FYSH560 spring 2011

Exercise 6, return by Tue Feb 1st at 14.15, discussed Wed March 2nd at 8.15 in FL140

1. Calculate
$$\int_{-\infty}^{\infty} \mathrm{d}k \frac{e^{ikx}}{k^2+m^2}.$$
This can be done the easy way using the theorem of residues; there is probably also a hard way without it. Then calculate the two dimensional transform needed in the virtual photon-dipole wave function
$$\int \mathrm{d}^2\mathbf{k}_T \frac{e^{i\mathbf{k}_T\cdot\mathbf{r}_T}}{\mathbf{k}_T{}^2+m^2}$$
and compare. This is perhaps easiest to do by integrating over the angle first, which leaves you with the Bessel $J_0(|\mathbf{k}_T||\mathbf{r}_T|) = J_0(kr)$. The integral over k then gives a modified Bessel K_0. Mathematica will do the k-integral for you, maybe even the two-dimensional one directly? Otherwise one probably needs integral representations of J_0 and K_0.

2. At HERA, a 27.5GeV electron beam collides with a 920GeV proton beam producing *diffractively* a J/Ψ-meson at $x_{\mathbb{P}} = 0.001$ and $Q^2 = M_{J/\Psi}^2 \approx 10\mathrm{GeV}^2$ with negligibly small t. What is the angle of the scattered J/Ψ with respect to the incoming proton direction? You can neglect the proton mass.

   One possible approach: from Q^2 and $M_{J/\Psi}^2$ you get β and $x = \beta x_{\mathbb{P}}$. Now go to light cone coordinates, where q has a large $-$-component; P and P' large $+$-components. You get q^- from P^+ and Q^2, then q^+ from the requirement that the outgoing electron momentum $k' = k - q$ has $k'^2 = 0$ and finally $\mathbf{q}_T^2$ from $q^2 = -Q^2$. Neglecting t and m_p means that $P^- = P'^- \approx 0$ and $P_\perp \approx 0$, thus the J/Ψ inherits q^- and $\mathbf{q}_T$ from the virtual photon (not q^+, why?). Knowing $p_{J/\Psi}^-, \mathbf{p}_T{}_{J/\Psi}^2$ and $M_{J/\Psi}$ gives you the rapidity and then the scattering angle via $p^z = \sqrt{\mathbf{p}_T{}^2 + m^2}\sinh y$. Where did the inelasticity y appear in the calculation?

3. Consider a system of 2 massless particles with the same transverse momentum $p_\perp$, in the same azimuthal direction. If the rapidities of the two particles are $y + \Delta/2$ and $y - \Delta/2$ calculate
   (a) The invariant mass of the two particle system
   (b) The total $+$ momentum of the system.

4. The Good-Walker picture of diffraction. Consider a hadronic state $|h\rangle$ scattering off a target (such as another hadron). The scattering is described by a T-matrix, $S = 1 + iT$. Assume that we have been able to diagonalize the T-matrix, i.e. to find the states $|n\rangle$ such that
$$\langle m|T|n\rangle = i\delta_{mn}A_n$$
Furthermore assume that the scattering amplitudes iA_n are imaginary, i.e. A_n real. The incoming state is
$$|h\rangle = \sum_n c_n|n\rangle$$
By the optical theorem the total cross section is
$$\sigma_{\mathrm{tot}} = 2\langle h|\mathrm{Im}T|h\rangle,$$
the elastic cross section
$$\sigma_{\mathrm{el}} = |\langle h|T|h\rangle|^2$$
Why is
$$\sigma_{\mathrm{diff}} = \sum_n |\langle n|T|h\rangle|^2 - \sigma_{\mathrm{el}}$$
the diffractive cross section? Express these cross sections in terms of the complex coefficients c_n and the amplitudes A_n.