FYSH560 spring 2011

Exercise 5, return by Tue Feb 22nd at 14.15, discussed Wed Feb 23rd at 8.15 in FL140

1. At HERA, a 27.5GeV electron beam collides with a 920GeV proton beam. In one event, $x = 0.1$ and $Q^2 = 50\text{GeV}^2$. In another, $x = 0.001$ and $Q^2 = 3\text{GeV}^2$. What is the scattering angle (with respect to the incoming electron beam) and energy of the outgoing electron in these events? You can neglect the proton mass.

2. In the target rest frame the electron-proton cross section is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E'\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{2mQ^2}\frac{E'}{E}L_{\mu\nu}W^{\mu\nu},$$

where the leptonic tensor is $L_{\mu\nu} = 2(k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k\cdot k')$ and the hadronic tensor

$$W_{\mu\nu} = -2\left(g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2}\right)F_1(x,Q^2) + \frac{2}{P\cdot q}\left[\left(P_\mu + \frac{P\cdot q}{Q^2}q_\mu\right)\left(P_\nu + \frac{P\cdot q}{Q^2}q_\nu\right)\right]F_2(x,Q^2)$$

Calculate $L_{\mu\nu}W^{\mu\nu}$ in terms of the Lorentz invariants $x, y, Q^2 = -q^2$ and m_N^2 (the electron is massless).

3. Let us introduce the projection operators to the virtual photon polarization states as

$$d_{\mu\nu} = -\left(g_{\mu\nu} + \frac{q^\mu q^\nu}{Q^2}\right) = -\varepsilon_\mu^{\mathrm{L}}(q)(\varepsilon_\nu^{\mathrm{L}}(q))^* + \sum_{\lambda=\pm1}\varepsilon_\mu^{(\lambda)}(q)(\varepsilon_\nu^{(\lambda)}(q))^* \qquad (1)$$

$$d_{\mu\nu}^{\mathrm{L}} = \frac{Q^2}{m^2(\nu^2+Q^2)}\left(P_\mu + \frac{P\cdot q}{Q^2}q_\mu\right)\left(P_\nu + \frac{P\cdot q}{Q^2}q_\nu\right) \qquad (2)$$

$$d_{\mu\nu}^{\mathrm{T}} = \frac{1}{2}\left(d_{\mu\nu} + d_{\mu\nu}^{\mathrm{L}}\right) \qquad (3)$$

These projectors have slightly unconventional properties from a mathematical point of view, due to the nonpositive Lorentz metric and the fact that we include the average over the two transverse spins in the projector. Note that there is a sign confusion in Barone & Predazzi. Write the explicit expressions (diagonal 4×4 matrices) of the projectors in the target rest frame $P^\mu = (m, \mathbf{0})$, $q^\mu = (\nu, \mathbf{0}_T, \sqrt{Q^2+\nu^2})$ and the brick wall frame $P^\mu = (m\sqrt{1+(\nu/Q)^2}, \mathbf{0}_T, -m\nu/Q)$, $q^\mu = (0, \mathbf{0}_T, Q)$.

4. Using the projectors and the decomposition of $W_{\mu\nu}$ given above express

$$\sigma_{\mathrm{L,T}}^{\gamma^* p} = \frac{4\pi^2\alpha_{\mathrm{e.m.}}}{Q^2(1-x)}x d_{\mu\nu}^{\mathrm{L,T}}W^{\mu\nu}$$

in terms of F_1 and F_2. Here we are (hopefully correctly) using the Hand convention for the flux factor $4m\nu(1-x)$ as in Halzen & Martin, differently from the Gilman convention in Barone & Predazzi. In the lectures we neglected the proton mass terms and the x in the flux factor, but they are easy to include here. Do you recover the expression used in eq. (10) of `hep-ex/9510009` (ZEUS collaboration).