FYSH560 spring 2011

Exercise 4, return by Tue Feb 15th at 14.15, discussed Wed Feb 16th at 8.15 in FL140

1. In the lecture we skipped over the proper color projector algebra. We have an amplitude with 4 external colored legs $A_{ij,lk}$. We project out the color singlet and octet parts using the projectors defined as

$$P_{\underline{1}kl}^{ij} = \frac{1}{N_c}\delta_{ij}\delta_{kl} \quad (1)$$

$$P_{\underline{8}kl}^{ij} = 2t^a_{ji}t^a_{lk} \quad (2)$$

$$(3)$$

Show that these are properly normalized projectors, i.e. $P_{\underline{1}kl}^{ij}P_{\underline{1}mn}^{lk} = P_{\underline{1}mn}^{ij}$, $P_{\underline{8}kl}^{ij}P_{\underline{8}mn}^{lk} = P_{\underline{8}mn}^{ij}$ and $P_{\underline{1}kl}^{ij}P_{\underline{8}mn}^{lk} = 0$. Compute the normalization $P_{\underline{1}kl}^{ij}P_{\underline{1}ji}^{lk}$ and $P_{\underline{8}kl}^{ij}P_{\underline{8}ji}^{lk}$. Now we want to decompose the color structure in the imaginary and real part of the one loop amplitude into color components as

$$\mathrm{Im}A \sim (t^b t^a)_{ji}(t^b t^a)_{lk} = \mathrm{I}_{\underline{1}}P_{\underline{1}kl}^{ij} + \mathrm{I}_{\underline{8}}P_{\underline{8}kl}^{ij}$$

and

$$\mathrm{Re}A \sim (t^b t^a)_{ji}[t^b, t^a]_{lk} = \mathrm{R}_{\underline{1}}P_{\underline{1}kl}^{ij} + \mathrm{R}_{\underline{8}}P_{\underline{8}kl}^{ij}.$$

Calculate $\mathrm{I}_{\underline{1}}$, $\mathrm{R}_{\underline{1}}$ and $\mathrm{R}_{\underline{8}}$. It is a bit more difficult to compute $\mathrm{I}_{\underline{8}}$, can you do it?

2. The Lipatov vertex (below) is $C^\rho = (C^+, C^-, \mathbf{C}_T) = (k_1^+ + \mathbf{k}_{T1}^2/k_2^-, k_2^- + \mathbf{k}_{T2}^2/k_1^+, -\mathbf{k}_{T1} - \mathbf{k}_{T2})$. (this has now the corrected sign). The outgoing gluon has momentum $p = k_1 - k_2$

   (a) Using the approximations in multi-Regge kinematics, express C^ρ in terms of p^+, p^- and $\mathbf{k}_{T1}, \mathbf{k}_{T2}$.

   (b) Remembering that $p^2 = 0$ show that $p^\rho C_\rho(k_1, k_2) = 0$

   (c) Calculate $C^\rho(k_1, k_2)C_\rho(k_1, k_2)$

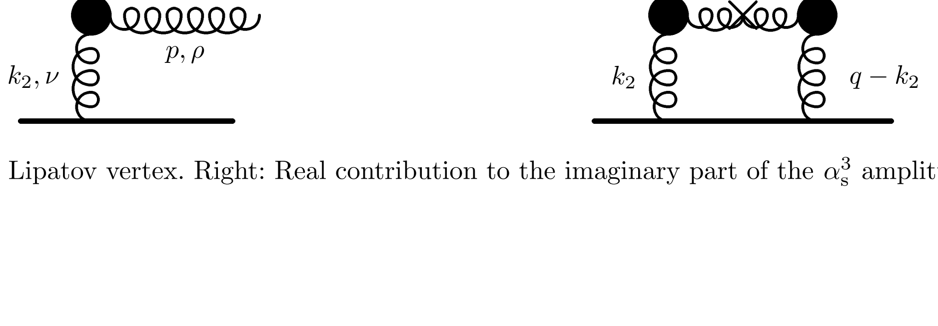
3. Calculate the transverse momentum integrand appearing in the real part of the α_s^3 amplitude:

$$\frac{C^\rho(k_1, k_2)C_\rho(q - k_1, q - k_2)}{\mathbf{k}_{T1}^2\mathbf{k}_{T2}^2(\mathbf{q}_T - \mathbf{k}_{T1})^2(\mathbf{q}_T - \mathbf{k}_{T2})^2}$$

4. Show how a Laplace transform deconvolutes the nested rapidity integrals in the BFKL ladder. I.e. if

$$f(y) = \int_0^y \mathrm{d}y_1 \int_0^{y_1} \mathrm{d}y_2 \cdots \int_0^{y_{n-1}} \mathrm{d}y_n e^{(y-y_1)(\varepsilon(k_1)+\varepsilon(q-k_1))} e^{(y_1-y_2)(\varepsilon(k_2)+\varepsilon(q-k_2))} \times \\ \cdots e^{(y_{n-1}-y_n)(\varepsilon(k_n)+\varepsilon(q-k_n))} e^{y_n(\varepsilon(k_{n+1})+\varepsilon(q-k_{n+1}))}$$

calculate the Laplace transform $f(\omega) = \int_0^\infty \mathrm{d}y e^{-\omega y} f(y)$. Hint: take the rapidity differences $y_n - y_{n+1}$ as integration variables.



Left: Lipatov vertex. Right: Real contribution to the imaginary part of the α_s^3 amplitude