FYSH560 spring 2011

Exercise 1, return by Tue Jan 25th at 14.15., discussed Wed Jan 26th, at 8.15 in FL140

1. The differential cross section for the $2 \to n$ process is

$$\mathrm{d}\sigma = \frac{1}{2s} \left| A(i \to f_n) \right|^2 \mathrm{d}\Pi_n.$$

   (a) What is the dimensionality (in GeV) of the scattering amplitude A for an arbitrary n?

   (b) Consider the following processes: $qq \to qq$, $gg \to gg$, $qq \to qqgg$ and $gg \to ggg$ (q is a quark and g is a gluon). Draw at least one Feynman diagram contributing to each, add upp the dimensionalities of the factors from the external lines, propagators and vertices and check that the amplitude has the right dimensions.

2. Let's practice SU(N_c) algebra. The fundamental representation is generated by the traceless Hermitian $N_\mathrm{c} \times N_\mathrm{c}$ matrices t_a, $a = 1 \dots N_\mathrm{c}^2 - 1$ normalized as $\mathrm{Tr} t_a t_b = \frac{1}{2}\delta_{ab}$. Their commutator is $[t_a, t_b] = i f_{abc} t_c$ with f_{abc} antisymmetric in all three indices (this actually follows from the chosen normalization). The fundamental representation Casimir operator is $t_a t_a = C_\mathrm{F} \mathbb{1}_{N_\mathrm{c} \times N_\mathrm{c}}$. What is C_F ? The adjoint representation is generated by $(T_a)_{bc} = -i f_{abc}$. The adjoint Casimir C_A in $f_{abc} f_{abd} = C_\mathrm{A} \delta_{cd}$ is $C_\mathrm{A} = N_\mathrm{c}$ (can you show this?). Using these calculate $\mathrm{Tr}(t_a t_b t_a t_b)$.

3. This calculation should sound familiar from the basic particle physics course. Calculate (to lowest order, one tree diagram) the differential QED cross section $\mathrm{d}\sigma/\mathrm{d}t$ for $e^-\mu^- \to e^-\mu^-$ scattering neglecting all the masses. Express the result in terms of the Mandelstam invariants s, t and u. What is the high energy limit of the cross section, i.e. the limit t fixed, $s \sim -u \to \infty$?

4. In the previous problem, what happens if you replace the photon by a massless scalar particle? I.e. replace $g_{\mu\nu} \to 1$ in the photon propagator and $\gamma^\mu \to 1$ in the vertex. What is the high energy limit now? (This calculation is shorter than the previous one).