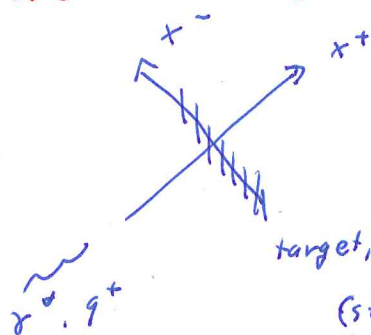


Collision of CYM fields

CYM = Classical Yang-Mills

We had a $x^\perp$ -target collision



any x^- -dependence is negligible compared to $e^{iq^+x^-}$ of probe.

Target is seen as static in x^- ; its own light cone time. In other words: the target is probed at a ~~short~~ timescale $\Delta x^- \sim \frac{1}{q^+}$ which is very short compared to its own natural timescale

$$\begin{aligned} \text{Timescale of target } \Delta x^- &\sim \frac{1}{p^+} \sim \frac{p^-}{m^2} \sim \frac{p^- q^+}{m^2} \frac{1}{q^+} \\ &\sim \frac{Q^2}{x m^2} \frac{1}{q^+} \\ &\gg 1 \end{aligned}$$

This kind of a system is called a gluon.

(A gluon is a liquid that flows so slowly that it looks like a solid on human timescales).

Class field $\leftarrow$ color current $j^-(x, x^+)$

$$[D_\mu, F^{\mu\nu}] = j^\nu \rightarrow A_a^-(x, x^+) = \frac{-1}{\partial_-^2} j_a^-(x, x^+)$$

LC wavef. / parton picture of target?

$\rightarrow$ target LC-gauge $A^- = 0$

$$A_\mu \rightarrow U A_\mu U^\dagger - \frac{i}{g} U \partial_\mu U^\dagger$$

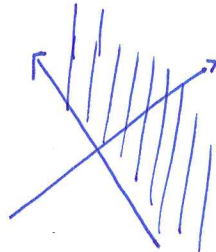
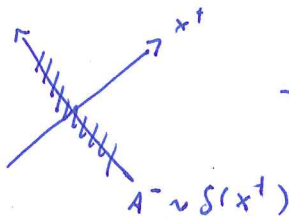
$$A^- \rightarrow U A^- U^\dagger - \frac{i}{g} U \partial_+ U^\dagger = 0 \rightarrow U^\dagger = \underline{P} e^{ig \int_{x^+}^{x^+} A^-(z^+, x)} \underline{U}$$

Same Wilson line as in DIS cross section.

Now we are in the target LC gauge. What are the new nonzero components?

$$A^+ : 0 \rightarrow -\frac{i}{g} U \partial_- U^\dagger \approx 0 \quad (\text{no } x^- \text{-dependence in static target})$$

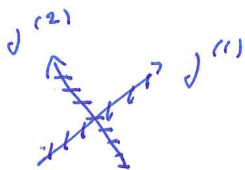
$$A_i : 0 \rightarrow -\frac{i}{g} U \partial_i U^\dagger = \text{pure gauge}$$



$A_i \sim \theta(x^+) \rightarrow$ the field is $\neq 0$, but
 $F_{\mu\nu} \sim \partial_+ \theta(x^+) \sim \delta(x^+)$
 $\rightarrow$ only lives on the light cone.

What about a collision of 2 such field configurations?
 Now the ~~field~~ field only has transverse components,
 so we can write down the fields for both targets.

$$J^\mu(x) = \delta^{\mu+} J^{(1)}(x, x^-) + \delta^{\mu-} J^{(2)}(x, x^+)$$



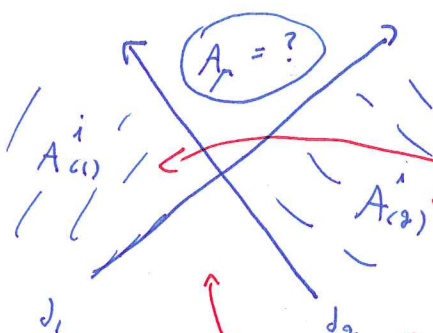
Both currents independent on their respective light cone times.

Form Wilson lines from both currents separately:

$$U_{(1)}^\dagger(x) = P \exp \left(i g \int_{-\infty}^{\infty} dx^- J^{(1)}(x, x^-) \right) \quad \leftarrow J^{(1)} \sim \delta(x^-)$$

$$U_{(2)}^\dagger(x) = P \exp \left(i g \int_{-\infty}^{\infty} dx^+ J^{(2)}(x, x^+) \right) \quad \leftarrow J^{(2)} \sim \delta(x^+)$$

} integrating directly to $\pm\infty$



$$A_{(m)}^i = \frac{i}{g} U_{(m)} \partial_i U_{(m)}^\dagger$$

pure gauge: field, but no energy density

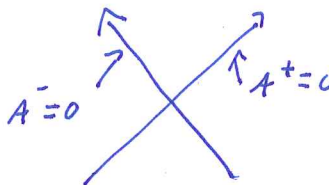
locally disconnected, no field

For $x^- > 0$, $x^+ > 0$ the field must be more complicated. This is the initial state that will evolve into a quark gluon plasma. In the classical field picture the field configurations are a solution of the Yang-Mills equation of motion

$[D_\mu, F^{\mu\nu}] = 0 \rightarrow$ nonlinear equation, not solvable ~~analytically~~ analytically. Cannot impose $A^+ = A^-$ any more, so one has to choose a less restrictive gauge condition:

$$A_\gamma = \frac{1}{2}(x^+ A^- + x^- A^+) = 0$$

$$\begin{aligned} \tau^+ &= 2x^+ x^- \\ \zeta &= \frac{1}{2} \ln \frac{x^+}{x^-} \end{aligned}$$

 $\rightarrow$ LC gauge on light cone.

The remaining orthogonal gauge field component is the y^- component:

$$A^\pm = \pm x^\pm A^z$$

$$\text{Ansatz: } A^i = \theta(x^-) \theta(-x^+) A_{(1)}^i + \theta(-x^-) \theta(x^+) A_{(2)}^i + \theta(x^+) \theta(x^-) A^i$$

$$A^\pm = \pm x^\pm \theta(x^+) \theta(x^-) A^z$$

$$[D_\mu, F^{\mu\nu}] = 0, \quad \partial_- \theta(x^-) = \delta(x^-), \quad \partial_+ \theta(x^+) = \delta(x^+)$$

Insert ansatz into equation, match coefficients of δ -functions to zero

$$\Rightarrow A^i(\tau=0^+) = A_{(1)}^i + A_{(2)}^i \rightarrow B^z \neq 0 \text{ (HT)}$$

$$A^z(\tau=0^+) = \frac{i g}{2} [A_{(1)}^i, A_{(2)}^i] \rightarrow E^z = F^{-+} = A^z(\tau=0)$$

$$B_\perp = E_\perp = 0 \quad (\tau=0^+)$$

These are the "glasma" fields (Kovner, McLerran, Weigert, -95;
 Iappi, McLerran, -06)

Initially longitudinal chromoelectric and -magnetic fields.

In the dilute limit they correspond to the field



configurations of
 a gluon produced
 in the $2 \rightarrow 1$
 mechanism with
 a dipole vertex

$$A^2 \sim \frac{1}{Y} J_1(k_\perp | r)$$

and unintegrated gluon distributions

$\langle S(1) S(1) \rangle$, $\langle S(2) S(2) \rangle$ from the
 two colliding color charged objects.

This is the end of the course. We have progressed
 from: ~~exp~~

Optical diffraction (19th century)

→ QM scattering theory

→ pre-QCD hadron scattering (Regge), -60's

→ QCD scattering - 70's,
 resummed by BFKL, -80's

→ dipole model, high energy DIS phenomenology, BK
 - 90's

→ classical field picture of heavy ion
 collisions, 2000's.