4-7

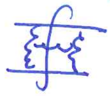
1.2.2011

Lipatov vertex

What about the next order in α_s? This should give us more insight into whether a pomeron / reggeized gluon structure is true at a higher order.

We will still want to calculate the next order corrections to leading order in s and using dispersive techniques. I.e. we will cut diagrams to obtain the imaginary part of the diagram that corresponds to physical on-shell particles in an intermediate state.

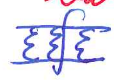
These cut diagrams are divided into 2 kinds of contributions:



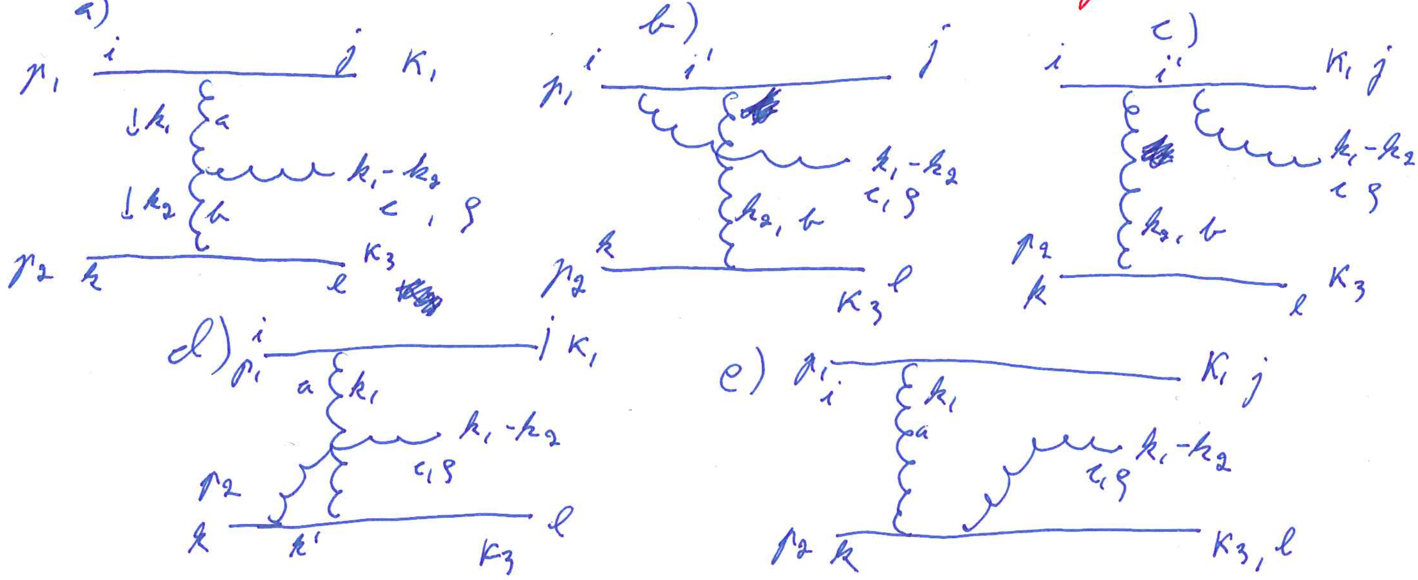
- "real": cut $qq+g$ →

N.B. Not 'real' as in Re/Im, but 'real' as in real vs. virtual = unreal

- "virtual": cut qq →



We concentrate first on the real contribution. We need the diagrams that produce $qq \to qqg$. There are 5 that contribute at leading $\ln s$:



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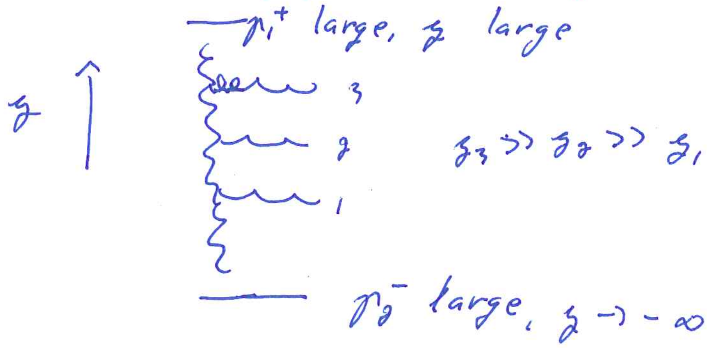
Sudakov:

$$k_{1,2} = (\alpha_{1,2}\, p_1^+ ,\ \beta_{1,2}\, p_2^- ,\ \underset{\sim}{k}_{1,2})$$

The leading contribution in the large s limit comes from the <u>multi-Regge</u> kinematical regime, where the rapidities of the gluons are strongly ordered.



$y_3 \gg y_2 \gg y_1$ $\hat{=}$ multi-Regge kinematics

$$\Rightarrow \quad p_1^+ \gg k_1^+ \gg k_2^+ \gg p_2^+ \ (=0)$$

$$p_2^- \gg k_2^- \gg k_1^- \gg p_1^- \ (=0)$$

$$\Rightarrow \quad \alpha_2 \ll \alpha_1 \ll 1$$

$$\beta_1 \ll \beta_2 \ll 1$$

Why is multi-Regge the dominant regime? Because all the "t-channel" propagators only have transverse momenta and are thus not suppressed by large momentum components $\sim \sqrt{s}$. In MRK all transverse momenta are assumed to be of the same order.

A practical result of this:

Produced gluon:

$$0 = (k_1 - k_2)^2 = 2(\alpha_1-\alpha_2)(\beta_1-\beta_2)\, p_1^+ p_2^- - (\underset{\sim}{k}_1 - \underset{\sim}{k}_2)^2$$

$$\alpha_1 \beta_2 s \approx (\underset{\sim}{k}_1 - \underset{\sim}{k}_2)^2$$

$$k_1^2 = \alpha_1 \beta_1 s - \underset{\sim}{k}_1^2 = \underbrace{\frac{\beta_1}{\beta_2}}_{\ll 1} \underbrace{(\underset{\sim}{k}_1 - \underset{\sim}{k}_2)^2}_{\text{same order of magnitude as } \underset{\sim}{k}_1} - \underset{\sim}{k}_1^2 \approx -\underset{\sim}{k}_1^2$$

$k_2^2 \approx -\underset{\sim}{k}_2^2 \to$ Regions of large α, β will give much larger $k_{1,2}^2$, which will suppress amplitude.

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$$A_a = (-i)(-2ig\,p_{1\mu})\,t^a_{ji}\left(\frac{-i}{k_1^2}\right)(-f_{abc})\times$$

$$\left[(k_1+k_2)^\rho g^{\mu\nu} + \underbrace{(k_1-2k_2)^\mu}_{p_1^+(k_1^- - 2k_2^-)\approx -2k_2^- p_1^+} g^{\nu\rho} + \underbrace{(k_2-2k_1)^\nu}_{p_2^-(k_2^+ - 2k_1^+)\approx -2p_2^- k_1^+} g^{\mu\rho}\right]\times$$

$$\left(\frac{-i}{k_2^2}\right)(-2ig\,p_{2\nu})\,t^b_{lk}$$

$$\approx \frac{+4ig^2 f_{abc}}{\underset{\sim}{k}_1^2\,\underset{\sim}{k}_2^2}\,p_{1\mu}p_{2\nu}\left[(k_1+k_2)^\rho g^{\mu\nu} - \underbrace{2k_2^\mu g^{\nu\rho}}_{(1)} - \underbrace{2k_1^\nu g^{\mu\rho}}_{(2)}\right]t^a_{ji}t^b_{lk}$$

(1) :
$$p_{1\mu}k_2^\mu = p_1^+k_2^- = \beta_2 p_1^+p_2^- = \beta_2 p_{1\mu}p_{2\nu}g^{\mu\nu}$$
$$p_{2\nu}g^{\nu\rho} = p_2^\rho$$

(2) :
$$p_{2\nu}k_1^\nu = p_2^-k_1^+ = \alpha_1 p_2^- p_1^+ = \alpha_1 p_{1\mu}p_{2\nu}g^{\mu\nu}$$
$$p_{1\mu}g^{\mu\rho} = p_1^\rho$$

$$= \frac{4ig^2 f_{abc}\,t^a_{ji}t^b_{lk}}{\underset{\sim}{k}_1^2\,\underset{\sim}{k}_2^2}\,p_{1\mu}p_{2\nu}g^{\mu\nu}\left[(k_1+k_2)^\rho - 2\beta_2 p_2^\rho - 2\alpha_1 p_1^\rho\right]$$

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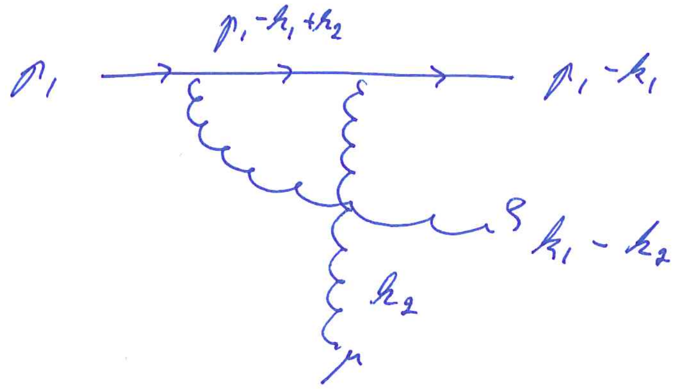
For diagram b, c, d and e we need a kind of a "double eikonal vertex" for the radiation of an additional gluon off the high energy quark line:

Suppressing colour indices:

$V_2 =$



$\not p \not p = p^2$, so $\not p \frac{\not p}{p^2} = 1$ and $\frac{1}{\not p} = \frac{\not p}{p^2}$

$$\underbrace{\bar u_\sigma(p_1-k_1)}_{\approx \bar u_\sigma(p_1)} \gamma^\mu \underbrace{\frac{i\,g^2}{\not p_1 - \not k_1 + \not k_2}} \gamma^\rho u_{\sigma'}(p_1)$$

$$= \frac{i(\not p_1 - \not k_1 + \not k_2)}{(p_1-k_1+k_2)^2} \approx \frac{i\not p_1}{\underbrace{(p_1-k_1+k_2)^2}_{\text{skalaaripropagaattori}}}$$

$$\not p_1 = \sum_\tau u_\tau(p_1)\bar u_\tau(p_1)$$

$$(p_1-k_1+k_2)^2 = \underbrace{p_1^2}_{=0} + \underbrace{(k_1-k_2)^2}_{=0} - 2p_1\cdot(k_2-k_1)$$

$$= -2p_1^+(\underbrace{k_2^-}_{\beta_2 p_2^-} - \underbrace{k_1^-}_{\text{small}}) \approx -s\beta_2$$

$$V_2 = \frac{-ig^2}{\beta_2 s}\sum_\tau \overbrace{\bar u_\sigma(p_1)\gamma^\mu u_\tau(p_1)}^{2p_1^\mu \delta_{\sigma\tau}} \overbrace{\bar u_\tau(p_1)\gamma^\rho u_{\sigma'}(p_1)}^{2p_1^\rho \delta_{\tau\sigma'}}$$

$$= \frac{ig^2}{\beta_2 s}(-2p_1^\mu i)(-2p_1^\rho i)\delta_{\sigma\sigma'}$$

kuin 2 eik. vertexiä ja skalaari-propagaattori välissä

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$$A_b = (-i)(-2p_1^\mu ig)\frac{i}{\beta_2 s}(-2p_1^\rho ig)\, t^b_{ji'} t^c_{i'i}\,(-2p_2^\nu ig)\frac{-ig_{\mu\nu}}{k_2^2} t^b_{ek}$$

$$A_c = (-i)(-2p_1^\rho ig)\frac{i}{\beta_2 s}(-2p_1^\mu ig)\, t^c_{ji'} t^b_{i'i}\,(-2p_2^\nu ig)\frac{-ig_{\mu\nu}}{k_2^2} t^b_{ek}$$

$$A_b + A_c = \frac{8p_1^\rho p_1^\mu p_2^\nu}{\beta_2 s\,(-k_2^2)}\underbrace{[t^b, t^c]_{ji}}_{= i f^{abc} t^a_{ji}}\, t^b_{ek}\, g_{\mu\nu}$$

(In the same way)

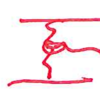
$$A_d + A_e = -i\frac{8p_2^\rho p_1^\mu p_2^\nu}{\alpha_1 s\, k_1^2} g_{\mu\nu} f^{abc} t^a_{ji} t^b_{ek}$$

Putting all together:

$$A_a + A_b + A_c + A_d + A_e$$

$$= -i\frac{\overbrace{(-2ig\,p_{1\mu})}^{\text{eik.}}\overbrace{(-2ig\,p_{2\nu})}^{\text{eik. vertex}}}{\underbrace{k_1^2\; k_2^2}_{\text{propagators}}} g^{\mu\nu}(-1)\Big[(k_1+k_2)^\rho - 2\beta_2 p_2^\rho \overbrace{-2\alpha_1 p_1^\rho}^{-2k_1^+}$$

$$-\frac{2k_1^2}{\beta_2 s}p_1^\rho - \frac{2k_2^2}{\alpha_1 s}p_2^\rho\Big]\underbrace{(-f^{abc})t^a_{ji} t^b_{ek}}_{\text{color factors}}$$

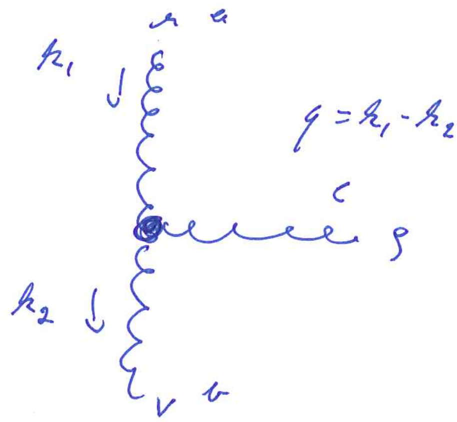
Just like , except 3-gluon vertex replaced by a more complicated structure; the Lipatov vertex.

Finally we arrive at an *effective vertex* that sums up the contribution of gluon radiation off the external (quark) legs of the diagram:

$$C^+ = -k_1^+ \underbrace{-k_2^+}_{\ll k_1^+} + \underbrace{2\alpha_1 p_1^+}_{2k_1^+} + \frac{\cancel{2}\mathbf{k}_1^2}{\cancel{2p_1^+}\underbrace{p_2^-\beta_2}_{=k_2^-}}\cancel{p_1^+} = k_1^+ + \frac{\mathbf{k}_1^2}{k_2^-}$$

$$C^- = k_2^- + \frac{\mathbf{k}_2^2}{k_1^+}$$

$$\mathbf{C} = \mathbf{k}_1 + \mathbf{k}_2$$



$$-f_{abc}\, g^{\mu\nu}\, C^\rho(k_1, k_2)$$

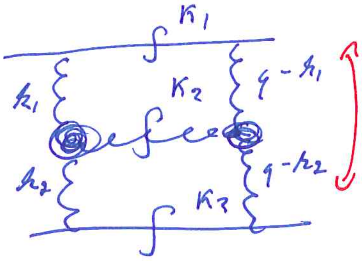
This is the *Lipatov vertex*. It is nonlocal due to the $\frac{1}{k_1^+}, \frac{1}{k_2^-}$ in the expression, which in position space correspond to inverse derivatives $\frac{1}{\partial_+}, \frac{1}{\partial_-}$.

Some properties:

- $q \cdot C = 0$ → "gauge invariant"
- $C^2 = -4\,\frac{\mathbf{k}_1^2 \mathbf{k}_2^2}{q^2}$

3.2.2011

3-particle phase space



$\Delta y, \quad s \sim e^{\Delta y}$

$$\int d\Omega_3 = \frac{(2\pi)^3}{(2\pi)^8} \int d^4k_1\, d^4k_2\, \delta((p_1-k_1)^2)\, \delta((p_2+k_2)^2)\, \delta((k_1-k_2)^2)$$

$$= \frac{s^2}{4(2\pi)^5} \int d^2\underset{\sim}{k}_1\, d^2\underset{\sim}{k}_2\, d\alpha_1\, d\alpha_2\, d\beta_1\, d\beta_2 \times$$

$$\delta(-\beta_1 \underbrace{(1-\alpha_1)}_{\approx 1} s - \underset{\sim}{k}_1^2)\, \delta(\alpha_2 \underbrace{(1+\beta_2)}_{\approx 1} s - \underset{\sim}{k}_2^2) \times$$

$$\delta(\underbrace{(\alpha_1-\alpha_2)(\beta_1-\beta_2)}_{-\alpha_1\beta_2} s - (\underset{\sim}{k}_1-\underset{\sim}{k}_2)^2)$$

$\to \int d\beta_2 : \beta_2 = -\frac{(\underset{\sim}{k}_1-\underset{\sim}{k}_2)^2}{\alpha_1 s} > -1$

$\Rightarrow \alpha_1 > \frac{(\underset{\sim}{k}_1-\underset{\sim}{k}_2)^2}{s}$

$$= \frac{1}{8\pi s} \int \frac{d^2\underset{\sim}{k}_1}{(2\pi)^2} \frac{d^2\underset{\sim}{k}_2}{(2\pi)^2} \underbrace{\int_{\frac{(\underset{\sim}{k}_1-\underset{\sim}{k}_2)^2}{s}}^{1} \frac{d\alpha_1}{\alpha_1}}$$

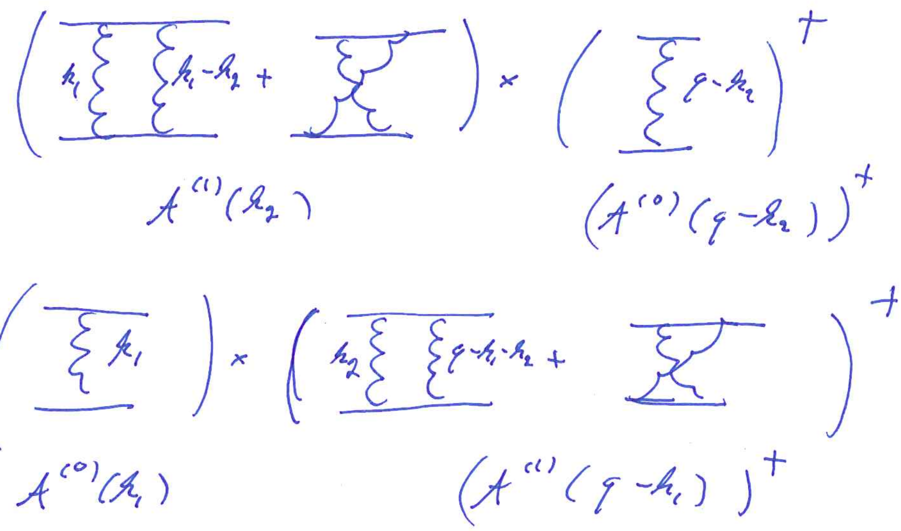
$\boxed{\sim \ln s}$ $\to$ this integral gives a "large logarithm" of s already in the Im part of the amplitude. Remember that $\mathrm{Im}(\Xi\Xi) \sim s$, no log. Also recall that $C^\mu(k_1, k_2)\, C_\mu(q-k_1, q-k_2)$ is transverse $\to$ does not affect this. Thus we expect $\ln^2 s$ in the real part.

This large logarithm comes from the fact that there is a large rapidity interval $\Delta y \sim \ln s$ and the extra gluon K_2 can be emitted (with a constant probability per unit rapidity) anywhere in this interval.

Virtual correction

We have now calculated (although we did not put together all the pieces) the real ($qq \to qqg$) contribution, expressed in terms of the Lipatov effective vertex. We will now discuss the other part; the virtual ($qq \to qq$) correction.

$2\,\mathrm{Im}\, A^{(2)}_{virt} =$



$$\mathrm{Im}\, A^{(2)}_{virt} = \frac{1}{2}\int d\Omega_2 \left[A^{(1)}(k_2)\left(A^{(0)}(q-k_2)\right)^+ + A^{(0)}(k_1)\left(A^{(1)}(q-k_1)\right)^+ \right]$$

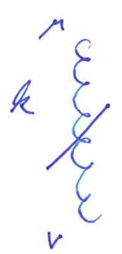
$$A^{(0)}(q-k_2) = 8\pi\alpha_s \frac{s}{(q-k_2)^2}\, t^a_{ji}\, t^a_{lk}$$

y, rapidity

$$A^{(1)}(k_2) = 8\pi\alpha_s \frac{s}{k_2^2} \ln\frac{s}{k_2^2}\, \varepsilon(k_2)$$

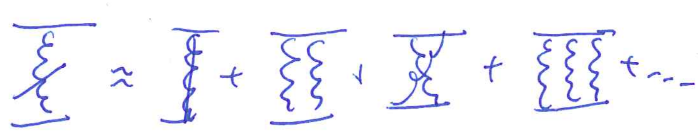
$$\varepsilon(k_2) = N_c \alpha_s \int \frac{d^2 k_1}{(2\pi)^2} \frac{-k_2^2}{k_1^2 (k_2 - k_1)^2}$$

These are equivalent (to this order in α_s) to the exchange of a reggeized gluon



$$\frac{-i g_{\mu\nu}}{k^2} e^{\Delta y\, \varepsilon(k^2)}$$

reggeized gluon propagator



Total cross section to α_s^3

We now have all the ingredients to compute the total cross section, both for the singlet and octet channels. We will not go through the considerable amount of algebra needed to put together all the pieces here; see Barone & Predazzi chap. 8 for details.

We just quote the result:

octet:

$$\mathrm{Im}\, A_{\underset{\sim}{8}}^{(2)} = \frac{N_c^2 \alpha_s^3}{2\pi^3} t^a_{ji} t^a_{lk}\, s \ln\frac{s}{|t|} \times \int d^2\underset{\sim}{k}_1\, d^2\underset{\sim}{k}_2 \frac{q^2}{\underset{\sim}{k}_1^2 \underset{\sim}{k}_2^2 (\underset{\sim}{k}_1 - \underset{\sim}{q})^2 (\underset{\sim}{k}_2 - \underset{\sim}{q})^2}$$

(0) + (1) + (2)

$$\mathrm{Im}\, A_{\underset{\sim}{8}} = 8\pi\alpha_s t^a_{ji} t^a_{lk} \left(\frac{s}{t}\right) \left[1 + \varepsilon(t) \ln\frac{s}{|t|} + \frac{1}{2}\varepsilon^2(t) \ln^2\frac{s}{|t|} \right]$$

singlet:

$$\mathrm{Im}\, A_{\underset{\sim}{1}}^{(2)} = -i \frac{2\alpha_s^3}{\pi^2} \frac{N_c^2 - 1}{4} \delta_{ij}\delta_{kl}\, s \ln\frac{s}{|t|} \int d^2\underset{\sim}{k}_1\, d^2\underset{\sim}{k}_2 \times$$

$$\left[\frac{q^2}{\underset{\sim}{k}_1^2 \underset{\sim}{k}_2^2 (\underset{\sim}{k}_1 - \underset{\sim}{q})(\underset{\sim}{k}_2 - \underset{\sim}{q})^2} - \frac{1}{2} \frac{1}{\underset{\sim}{k}_2^2 (\underset{\sim}{k}_1 - \underset{\sim}{q})^2 (\underset{\sim}{k}_1 - \underset{\sim}{k}_2)^2} - \frac{1}{2} \frac{1}{\underset{\sim}{k}_1^2 (\underset{\sim}{k}_2 - \underset{\sim}{q})^2 (\underset{\sim}{k}_1 - \underset{\sim}{k}_2)^2} \right]$$

These similar terms are there also in the octet calculation, but they cancel between the real and virtual terms. In the singlet they do not, because of the different color factors.

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BFKL ladder

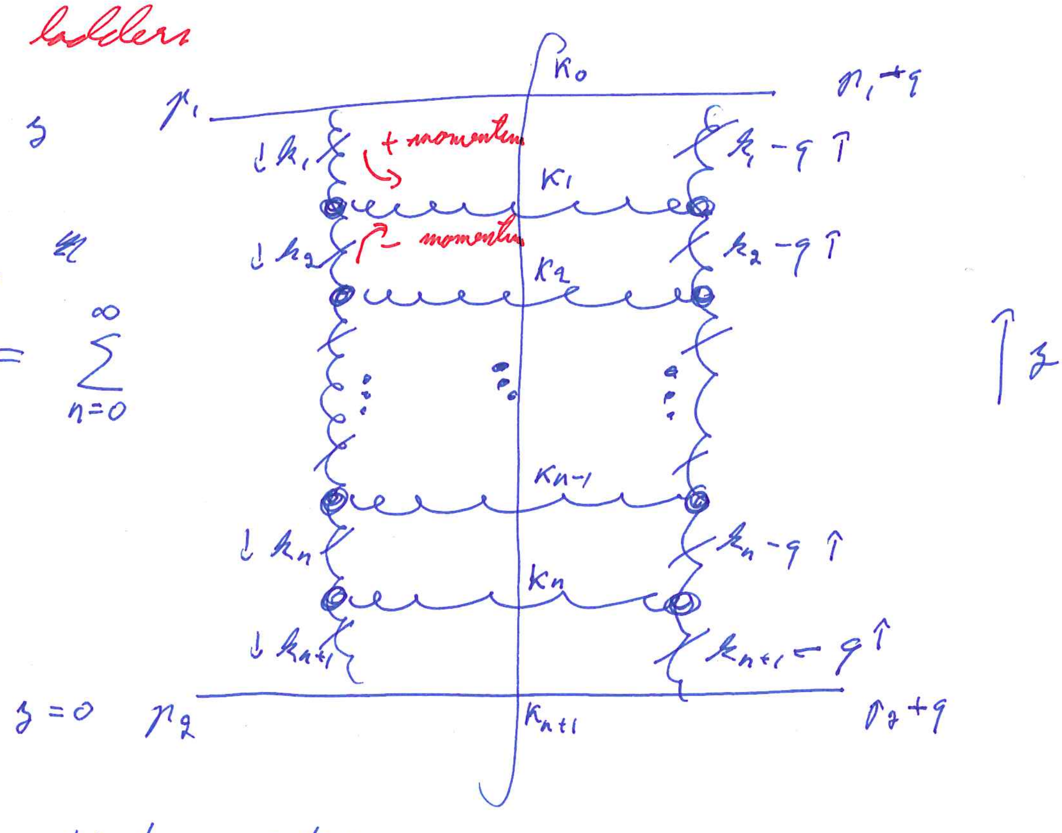
Balitsky, Fadin, Kuraev, Lipatov

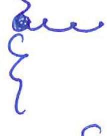
It is possible to resum all the dominant diagrams at leading $\ln s$.

Power counting: $\alpha_s \ll 1$, $\alpha_s \ln s \sim 1$

At each new order in α_s only include terms that come with an additional $\ln s$ to compensate it. Result: the dominant contribution is from ladders

$\mathrm{Im}\, A_{qq\to qq} \approx \sum_{n=0}^{\infty}$




Lipator vertex


reggeized propagator

Sudakov: $k_i = (\alpha_i p_1^+, \beta_i p_2^-, \underset{\sim}{k}_i)$

MRK:

$\alpha_1 \gg \alpha_2 \gg \dots \gg \alpha_{n+1}$

$\beta_1 \ll \beta_2 \ll \dots \ll \beta_{n+1}$

Momentum flow:

$K_i^+ \approx k_i^+ \sim e^{+y_i}$

$K_i^- \approx k_{i+1}^- \sim e^{-y_i}$

$\Rightarrow \xi > y_1 > y_2 > \dots > y_n > 0$

rapidity ordered cascade

Compare: DGLAP is transverse momentum ordered, dominant when $-q^2 = -t$ is large.

The phase space of the BFKL ladder is that of $n+2$ particles. This sounds complicated, but we can simplify it a lot in MRK.

$$d\Omega_{n+2} = \prod_{i=0}^{n+1}\left[\frac{d^4k_i}{(2\pi)^4}\,2\pi\delta(k_i^2)\right](2\pi)^4\delta^4(p_1+p_2-\sum_i k_i)$$

rapidity

- $\int dk_i^- dk_i^+ \,\delta(2k_i^- k_i^+ - \underset{\sim}{k}^2) = \int \frac{dk_i^+}{2k_i^+} = \frac{1}{2}\int dy_i$

- $\delta(p_1^+ - \sum_i k_i^+)\,\delta(p_2^- - \sum_i k_i^-) \approx \delta(p_1^+ - k_0^+)\,\delta(p_2^- - k_{n+1}^-)$

(remember MRK!)

~~$\Rightarrow d\Omega_{n+2}$~~

- $\underbrace{\prod_{i=0}^{n+1}\frac{d^2\underset{\sim}{k}_i}{(2\pi)^2}}_{n+2}\underbrace{(2\pi)^2\delta^2(\sum_{i=0}^{n+1}\underset{\sim}{k}_i)}_{-1} = \underbrace{\prod_{i=1}^{n+1}\frac{d^2\underset{\sim}{k}_i}{(2\pi)^2}}_{n+1}$

$$d\Omega_{n+2} = (2\pi)^2\prod_{i=1}^{n+1}\frac{d^2\underset{\sim}{k}_i}{(2\pi)^2}\;\underbrace{\prod_{i=1}^{n}\frac{1}{4\pi}\int dy_i}_{\sim(\ln s)^n}$$

$\sim(\ln s)^n$ → ~~each~~ every rung brings $\alpha_s \ln s$

MRK limits

$$A(y) \sim \int_0^y dy_1 \int_0^{y_1} dy_2 \cdots \int_0^{y_{n-1}} dy_n\; e^{(y-y_1)(\varepsilon(k_1)+\varepsilon(q-k_1))} \times$$

$$\cdots e^{(y_n-0)(\varepsilon(k_{n+1})+\varepsilon(q-k_{n+1}))}$$

Deconvolute with Laplace $\int_0^\infty dy\, e^{-\omega y} A(y)$

Laplace in y = Mellin transform in s

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Recall that the expression for the amplitude (Im part) was a sum over $n = 0 \dots \infty$ of n-rung ladders, each with a $2(n+1)$-dimensional transverse momentum integral.

After the Laplace-transform this can be expressed by a recursive formula.

R = singlet / octet

We define the intermediate quantity

$$F_R(\omega; \underset{\sim}{k}_1; q) = \frac{1 - C_R \int d^2\underset{\sim}{k}_2 \, \frac{K(\underset{\sim}{k}_1; \underset{\sim}{k}_2)}{\underset{\sim}{k}_2^2 (q - \underset{\sim}{k}_2)^2} F_R(\omega; \underset{\sim}{k}_2; q)}{\omega \underbrace{- \varepsilon(-\underset{\sim}{k}_1^2)}_{-\varepsilon_1} - \varepsilon(-(\underset{\sim}{k}_1 - q)^2)}$$

BFKL This is the general form with different coefficients for different channels R.

$$\mathrm{Im}\, A_R(\omega, q) = G_R \int d^2\underset{\sim}{k}_1 \, \frac{F_R(\omega; \underset{\sim}{k}_1; q)}{\underset{\sim}{k}_1^2 (q - \underset{\sim}{k}_1)^2}$$

K_{12} = 

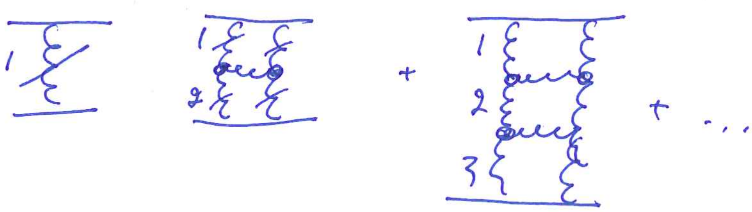
$C_R, \; G_R \sim \alpha_s$

Def. $\int_1 \equiv \int \frac{d^2\underset{\sim}{k}_1}{\underset{\sim}{k}_1^2 (q - \underset{\sim}{k}_1)^2} \quad \to \quad F_1 = \frac{1 - \int_2 K_{12} F_2}{\omega - \varepsilon_1}$

$$\mathrm{Im}\, A \sim \int_1 F_1 = \int_1 \frac{1 - \int_2 K_{12} F_2}{\omega - \varepsilon_1} = \int_1 \frac{1}{\omega - \varepsilon_1} - \int_{12} \frac{K_{12}}{\omega - \varepsilon_1} F_2$$

$$= \int_1 \frac{1}{\omega - \varepsilon_1} - \int_{12} \frac{K_{12}}{(\omega - \varepsilon_1)(\omega - \varepsilon_2)} \left(1 - \int_3 K_{23} F_3\right) + \dots$$

$F_3 = \frac{1}{\omega - \varepsilon_3} - \dots (\dots)$



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Let us try to give some physical interpretation to this equation.

$$\underset{\substack{\uparrow \\ \frac{\partial}{\partial y}}}{\omega F_1} = \underset{\substack{\uparrow \\ \text{init. cond} \\ @\ y=0}}{1} - \underset{\substack{\nwarrow \\ \text{real}}}{\int_2 K_{12} F_2} + \underset{\substack{\uparrow \\ \text{virtual}}}{\Sigma_1 F_1}$$

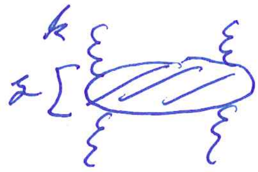
$\omega \leftarrow$ laplace

The BFKL equation is often encountered as a differential-integral evolution equation in rapidity

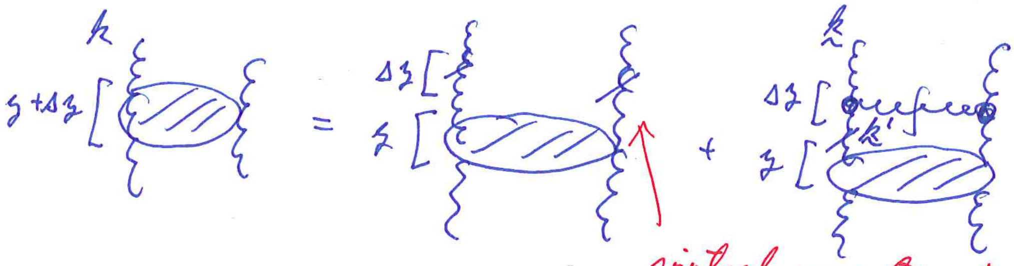
$$\frac{d}{dy} F_y(k) = \int_k \left[K_{real}(k,k') F_y(k') + K_{virt}(k,k') F_y(\overset{\text{not } k'}{k}) \right]$$

(Suppressing q, color indices)

This corresponds to the change in the scattering amplitude as a gluon distribution inside a hadron when the rapidity / energy is increased by Δy.

$F_y(k) =$ 

$$F_{y+\Delta y}(k) = F_y(k)\, e^{\Delta y\, \varepsilon(k)} + \Delta y \int_{k'} K(k,k')\, F_y(k')$$



$K \sim C^2$, square of Lipatov vertex

virtual correction / reggeized gluon is a Sudakov factor, makes sure probability is conserved.

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Solutions

- octet: easy → confirm reggeized gluon

$$A_g = -2\pi\alpha_s\left[1 - e^{-i\pi(1+\varepsilon(t))}\right]\left(\frac{s}{|t|}\right)^{1+\varepsilon(t)}$$

- singlet ("QCD pomeron"). Equation more complicated.
  - eigenfunctions of kernel needed in order to solve it. These can be found, but there is a considerable calculation to do it; takes a few pages in textbooks.
  - result: $A_1 \sim s^{1+\frac{N_c\alpha_s}{\pi}4\ln 2} \approx s^{1+0.5}$
    - analytical structure not quite Regge (Instead of a pole in the l-plane there is a branch cut.) Thus the exact form is not quite as simple as the Regge parametrisation.

$0.5 > 0.08$

Phenomenologically this exercise is a partial success.

+ made connection to successful Regge parametrization
- energy dependence is much faster than $s^{1.08}$ seen in data.
  → weak coupling QCD doesn't quite describe total cross sections ($t=0$!)
+ will work better at higher $|t|$ and be basis for further development.