FYSH555, spring 2014

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Part 8: Classical color fields

1 Eikonal propagation in target color field

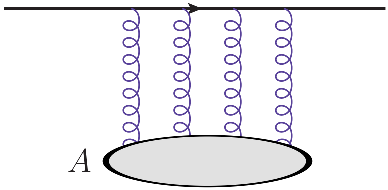
What is the target made of?

- So far we have not specified anything about the degrees of freedom in the target.
- We will argue that at high energy the target consists dominantly of gluons
  - We know that at small x the gluon distribution is larger than the quark one.
  - BK equation builds up the target by adding **gluons** to it.

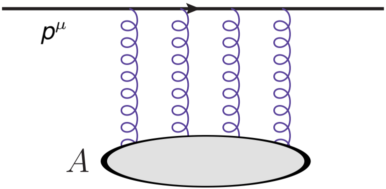
Color Glass Condensate (CGC)

We assume that there are so many gluons in the target, that it can be described by a **classical** gluon field. This is the heart of the CGC effective theory.

Many gluons = large color field A_μ
Have to sum all diagrams with n gluons lines
— but we can assume the gluons are a classical field



What is the target made of?



Quark propagating in classical color field: Dirac equation!

$$(i\partial\!\!\!/ - gA\!\!\!/)\psi(x) = 0$$

(Note: $A\!\!\!/ = A_a^\mu \gamma_\mu t^a$ is $N_c \times N_c$-matrix)

Want to dig out the dominant contribution: **eikonal** approximation

- Gluon is spin 1: it couples to a vector: $\sim p^\mu A_\mu$
- For high energy particle the only momentum available is p^μ
- p^μ has one large component: $p^+ \implies p^\mu A_\mu \sim p^+ A^- \implies$ only need A^-

Ansatz for DE: $\psi(x) = V(x)e^{-ip\cdot x}u(p)$, plug into equation $N_c \times N_c$**-matrix!**

$$\implies \quad \partial_+ V(x^+, x^-, \mathbf{x}) = -igA^-(x^+, x^-, \mathbf{x})V(x^+, x^-, \mathbf{x})$$

This is solved by path-ordered exponential

$$V(x^+, x^-, \mathbf{x}) = \mathbb{P}\exp\left\{-ig\int^{x^+} dy^+ A^-(y^+, x^-, \mathbf{x})\right\}$$

Eikonal propagation

- Now we know how a high energy quark propagates in a classical field.
- Thus we know the scattering S-matrix element for many-quark states

E.g. incoming free quark $|q_i(\mathbf{x})\rangle$ at $x^+ \to -\infty$ is, at $x^+ \to \infty$

$$|q_i(\mathbf{x})\rangle_{\text{in}} = \left[\mathbb{P}\exp\left\{-ig\int_{-\infty}^{\infty} \mathrm{d}y^+ A^-(y^+, x^-, \mathbf{x})\right\}\right]_{ji} |q_j(\mathbf{x})\rangle_{\text{out}}$$

a linear superposition of color rotated outgoing quarks.

- In scattering problem integrate $x^+ \in [-\infty, \infty]$
- In the high energy limit quark wavefunction oscillates like $e^{ip^+x^-}$ with large p^+ $\implies$ x^--dependence negligible compared to this $\implies$ approximate $x^- = 0$

Scattering is described by 2-dimensional field of SU(N_c)-matrices

$$V(\mathbf{x}) \equiv \mathbb{P}\exp\left\{-ig\int_{-\infty}^{\infty} \mathrm{d}x^+ A^-(x^+, x^- = 0, \mathbf{x})\right\}$$

— These is known as the **Wilson lines**

Dipole amplitude and Wilson lines

Incoming dipole (color neutral, average over colors!) changes into

$$|\mathrm{in}\rangle = \frac{\delta_{ii'}}{N_c} |q_i(\mathbf{x})\bar{q}_{i'}(\mathbf{y})\rangle_{\mathrm{in}} = \frac{\delta_{ii'}}{N_c} V_{ji}(\mathbf{x}) V^\dagger_{i'j'}(\mathbf{y}) |q(\mathbf{x})_j \bar{q}(\mathbf{y})_{j'}\rangle_{\mathrm{out}} \quad (V(\mathbf{y})^\dagger_{jk} = V(\mathbf{y})^*_{kj} \text{ for antiquark})$$

The total cross section is related to the imaginary part of the **forward elastic scattering amplitude**; i.e. we need to count outgoing dipoles in this state

$$S = {}_{\mathrm{out}}\langle q_k(\mathbf{x})\bar{q}_k(\mathbf{y})|\,\mathrm{in}\rangle = \frac{\delta_{ii'}}{N_c}\delta_{kj}\delta_{kj'} V_{ji}(\mathbf{x}) V^\dagger_{i'j'}(\mathbf{y}) = \frac{1}{N_c}\mathrm{Tr}\, V(\mathbf{x}) V^\dagger(\mathbf{y})$$

Dipole amplitude in the CGC

Relate $\mathcal{N}$ in BK and DIS to a **microscopical description of the target**:

$$\mathcal{N}_{q\bar{q}} = 1 - \frac{1}{N_c}\mathrm{Tr}\, V(\mathbf{x}) V^\dagger(\mathbf{y})$$

Note conventions

$$S_{fi} = \langle f|\,\hat{S}\,|f\rangle = 1 + iT_{fi} \quad \sigma_{\mathrm{tot}} = 2\mathrm{Im}\, T_{ii} \quad \mathcal{N} \equiv \mathrm{Im}\, T_{ii} \quad S_{ii} = \delta_{ii}(1-\mathcal{N}) + \mathrm{imag}$$

More complicated operators

- The dipole amplitude is a target expectation value of a two-point function

$$\mathcal{N}_{q\bar{q}} = 1 - \left\langle \hat{D} \right\rangle = \left\langle 1 - \frac{1}{N_c} \operatorname{Tr} V(\mathbf{x}) V^\dagger(\mathbf{y}) \right\rangle_{\text{target}}$$

- For this we derived the BK equation using a **mean field** approximation $\left\langle \hat{D}\hat{D} \right\rangle \approx \left\langle \hat{D} \right\rangle \left\langle \hat{D} \right\rangle$
- Similarly define other correlators, such as $\left\langle \hat{D}\hat{D} \right\rangle$ or the quadrupole

$$Q = \left\langle \frac{1}{N_c} \operatorname{Tr} V(\mathbf{x}) V^\dagger(\mathbf{y}) V(\mathbf{u}) V^\dagger(\mathbf{v}) \right\rangle_{\text{target}},$$

  and the corresponding evolution equations.
- Without the mean field approx. these operators couple to each other (e.g. $\partial_y \left\langle \hat{D} \right\rangle \sim \left\langle \hat{D}\hat{D} \right\rangle$) the **Balitsky hierarchy** of evolution equations
- The hierarchy can be generalized into an evolution equation for the **probability distribution of Wilson lines** — the JIMWLK equation

From BK to JIMWLK

JIMWLK equation

Gives rapidity-dependence of probability distribution of Wilson lines

$$\partial_y W_y[U(\mathbf{x})] = \mathcal{H} W_y[U(\mathbf{x})]$$

$$\mathcal{H} \equiv \frac{1}{2} \int_{\mathbf{xyz}} \frac{\delta}{\delta \widetilde{A}_c^+(\mathbf{y})} \mathbf{e}^{ba}(\mathbf{x}, \mathbf{z}) \cdot \mathbf{e}^{ca}(\mathbf{y}, \mathbf{z}) \frac{\delta}{\delta \widetilde{A}_b^+(\mathbf{x})},$$

$$\mathbf{e}^{ba}(\mathbf{x}, \mathbf{z}) = \frac{1}{\sqrt{4\pi^3}} \frac{\mathbf{x} - \mathbf{z}}{(\mathbf{x} - \mathbf{z})^2} \Big(1 - U^\dagger(\mathbf{x}) U(\mathbf{z})\Big)^{ba}$$

You can derive this in a very similar way as we did for BK.

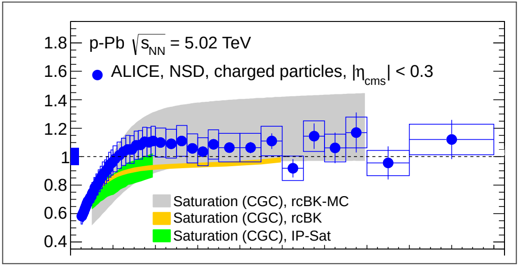
- Assume there is a y-dependent probability distribution $W_y[U(\mathbf{x})]$
- Consider collection of n Wilson lines propagating through target
- Emit one extra soft gluon and absorb small-z divergence into redefinition of probability distribution: $W_y[U(\mathbf{x})] \to W_{y+\Delta y}[U(\mathbf{x})]$

2 Particle production in proton-nucleus (pA) collisions

Nuclear modification factor R_{pA}

ALICE data on particle production in pA and pp & some predictions

(R_{pA} is ratio of cross sections in pA vs pp, normalized by geometry)



There are two ways to calculate this in the CGC

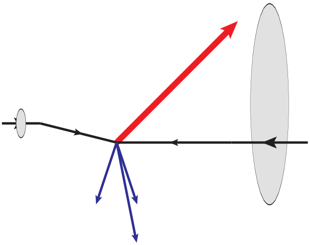
k_T-factorization Good at midrapidity/symmetric situation with strong color fields in **both** colliding objects. This we will come to a bit later

Hybrid formalism One colliding object described as dilute collection of partons $\Longrightarrow$ good at forward rapidity. First this.

Dilute-dense scattering

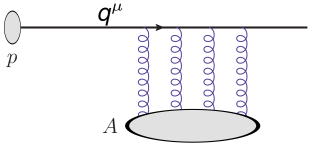
Look at forward rapidity pA

- The produced particle has large p^+.
- Momentum conservation: it comes from large x parton in proton
- At large x the proton is dilute collection of valence quarks
  $\Longrightarrow$ quark scattering on dense target



In: quark with momentum $q^+, \mathbf{q}$, color i

$$|\text{in}\rangle = \int d^2\mathbf{x} e^{-i\mathbf{q}\cdot\mathbf{x}} \, |q_i(\mathbf{x})\rangle_{\text{in}}$$

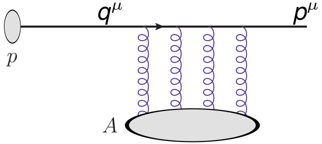


After interaction with the target

$$|\text{in}\rangle = \int d^2\mathbf{x} e^{-i\mathbf{q}\cdot\mathbf{x}} V_{ji}(\mathbf{x}) \, |q(\mathbf{x})_j\rangle_{\text{out}}$$

Scattering amplitude

$$|\text{in}\rangle = \int \mathrm{d}^2\mathbf{x} e^{-i\mathbf{q}\cdot\mathbf{x}} V_{ji}(\mathbf{x}) \, |q(\mathbf{x})_j\rangle_{\text{out}}$$



Scattering amplitude by projecting quarks with momentum **p** in the final state

(Neglect the 1 $\Longrightarrow \delta^2(\mathbf{q}-\mathbf{p})$ in $S = 1 + iT$)

$$\mathcal{M}_{i,\mathbf{q}\to k,\mathbf{p}} = {}_{\text{out}}\langle q_k(\mathbf{p})|\,\text{in}\rangle = \int \mathrm{d}^2\mathbf{x}\,\mathrm{d}^2\mathbf{y} e^{-i(\mathbf{q}\cdot\mathbf{x}-\mathbf{p}\cdot\mathbf{y})} V_{ji}(\mathbf{x}) \, \overbrace{{}_{\text{out}}\langle q_k(\mathbf{y})|\; q(\mathbf{x})_j\rangle_{\text{out}}}^{\delta^2(\mathbf{y}-\mathbf{x})\delta_{kj}}$$

We can choose $\mathbf{q} = 0$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\mathbf{p}} = \frac{1}{N_c}\frac{1}{(2\pi)^2}\sum_{i,k} |\mathcal{M}_{i,\mathbf{q}\to k,\mathbf{p}}|^2 = \frac{1}{N_c}\frac{1}{(2\pi)^2}\int \mathrm{d}^2\mathbf{x}\,\mathrm{d}^2\mathbf{y} e^{-i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})}\,\mathrm{Tr}\, V(\mathbf{x})V^\dagger(\mathbf{y})$$

There are $xq(x,\mu^2)$ incoming quarks in the proton per unit rapidity.

Hybrid formula for quark production

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y} = \frac{1}{(2\pi)^2} xq(x,\mu^2)\frac{1}{N_c}\int \mathrm{d}^2\mathbf{x}\,\mathrm{d}^2\mathbf{y} e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})}\,\mathrm{Tr}\, V(\mathbf{x})V^\dagger(\mathbf{y})$$

Normalization more carefully: cross section in LCPT

- Particle number operator

$$\frac{d\hat{N}}{d^3\vec{k}} = a^\dagger_{\vec{k}} a_{\vec{k}} \implies \frac{d\hat{N}}{d^3\vec{k}}|\vec{q}\rangle = a^\dagger_{\vec{k}} a_{\vec{k}} a^\dagger_{\vec{q}}|0\rangle = \delta^{(3)}(\vec{k}-\vec{q})|\vec{q}\rangle$$

- State normalization:

$$\langle\vec{k}|\vec{q}\rangle = \delta^{(3)}(\vec{k}-\vec{q}) \implies \langle\vec{k}|\vec{k}\rangle = \frac{S_\perp L^-}{(2\pi)^3} \implies \text{normalized 1-part. state } \frac{\sqrt{(2\pi)^3}}{\sqrt{S_\perp L^-}}|\vec{q}\rangle$$

(L^-: size of box in x^--direction: $\delta(k^+ = 0) = \int \frac{x^-}{2\pi} e^{ix^- \cdot 0} = L^-/(2\pi)$)

- Cross section

$$d\sigma = \frac{dN_{\text{out}}}{dt \cdot \frac{N_{\text{in}}=1}{dt S_\perp}} = S_\perp \frac{(2\pi)^3}{L^- S_\perp} {}_{\text{in}}\langle\vec{q}|\, d\hat{N}_{\text{out}} |\vec{q}\rangle_{\text{in}}$$

- Position eigenstates

$$|\mathbf{x}, k^+\rangle = \int \frac{d^2\mathbf{k}}{\sqrt{(2\pi)^2}} e^{-i\mathbf{k}\cdot\mathbf{x}} |\vec{k}\rangle$$

Normalization more carefully II

- Eikonal scattering

$$|\vec{q}, i\rangle_{\text{in}} = \int \frac{\mathrm{d}^2\mathbf{x}}{\sqrt{(2\pi)^2}} e^{i\mathbf{q}\cdot\mathbf{x}} |\mathbf{x}, q^+, j\rangle_{\text{in}} = \int \frac{\mathrm{d}^2\mathbf{x}}{\sqrt{(2\pi)^2}} e^{i\mathbf{q}\cdot\mathbf{x}} V_{ji}(\mathbf{x}) |\mathbf{x}, q^+, j\rangle_{\text{out}}$$

$$= \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \underbrace{\int \mathrm{d}^2\mathbf{x} e^{i(\mathbf{q}-\mathbf{k})\cdot\mathbf{x}} V_{ji}(\mathbf{x})}_{\equiv V_{ji}(\mathbf{q}-\mathbf{k})} |\mathbf{k}, q^+, j\rangle_{\text{out}}$$

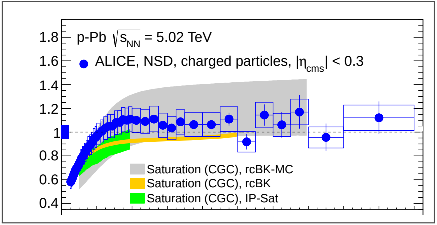
- Single inclusive cross section in eikonal scattering

$$\frac{\mathrm{d}\sigma^{\vec{q}+A\to\vec{p}+X}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}p^+} = \frac{(2\pi)^3}{L^-} {}_{\text{in}}\langle\vec{q}, i| \frac{\mathrm{d}\hat{N}_{\text{out}}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}p^+} |\vec{q}, i\rangle_{\text{in}} = \frac{(2\pi)^3}{L^-} \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \frac{\mathrm{d}^2\mathbf{k}'}{(2\pi)^2}$$

$$\times \operatorname{Tr} U^\dagger(\mathbf{q}-\mathbf{k}) U(\mathbf{q}-\mathbf{k}') \overbrace{{}_{\text{out}}\langle\mathbf{k}', q^+| \frac{\mathrm{d}\hat{N}_{\text{out}}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}p^+} |\mathbf{k}, q^+\rangle_{\text{out}}}^{\frac{L^-}{2\pi}\delta^2(\mathbf{p}-\mathbf{k})\delta^2(\mathbf{p}-\mathbf{k}')\delta(q^+-p^+)}$$

$$= \frac{\delta(q^+-p^+)}{(2\pi)^2} \int \mathrm{d}^2\mathbf{x}\,\mathrm{d}^2\mathbf{y} e^{i(\mathbf{q}-\mathbf{p})\cdot(\mathbf{x}-\mathbf{y})} \operatorname{Tr} U^\dagger(\mathbf{x}) U(\mathbf{y})$$

Back to R_{pA}



$$\frac{d\sigma}{d^2\mathbf{q}\,dy} = \frac{1}{(2\pi)^2} xq(x,\mu^2)\frac{1}{N_c}\int d^2\mathbf{x}\, d^2\mathbf{y} e^{-i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})}\,\mathrm{Tr}\, V(\mathbf{x})V^\dagger(\mathbf{y})$$

Now all we need is a parametrization, for protons **and** nuclei of

$$\mathrm{Tr}\, V(\mathbf{x})V^\dagger(\mathbf{y})$$

- Fit to HERA DIS data $\Longrightarrow$ proton dipole amplitude
  - using BK equation (remember: BK gives x-dependence, need to fit initial condition)
  - or some other model of the dipole cross section
- Generalize to nuclei: somehow incorporate Woods-Saxon $T_A(b)$
- The HERA data is very precise and theory fits it well: the “theory errors” in the above plot are all from this proton $\Longrightarrow$ nucleus generalization.

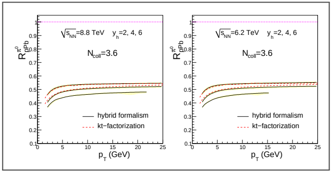
From protons to nuclei

One typical initial condition for BK: GBW Golec-Biernat, Wusthoff :

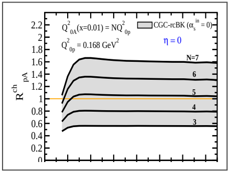
$$\mathcal{N}(\mathbf{b},\mathbf{r}) = \theta(R_p - b)\left(1 - \exp\left\{-\frac{\mathbf{r}^2}{4Q_s^2}\right\}\right), \qquad \text{and for nucleus?}$$

1. Just fit Q_s^A separately to some nuclear data
2. Assume saturation scale $Q_s^2 \sim T_A(\mathbf{b})$ or $A^{1/3}$ — with what coefficient?
3. Monte Carlo Glauber nucleus: count nucleons, $\left(Q_s^A\right)^2 = N_N \left(Q_s^A\right)^2$
   — Fine, but what is the area of the nucleon when you calculate N_N? Same as in DIS? Same as in Glauber? (These are different!)

One has to be careful (I'm being nasty showing these celebrated plots)



Oops!



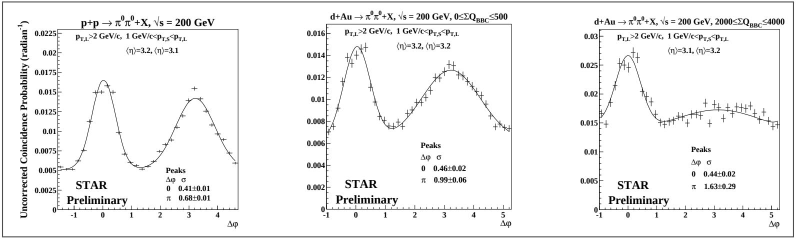
And the prediction was?

Differences mostly in nuclear geometry, not in the QCD part!

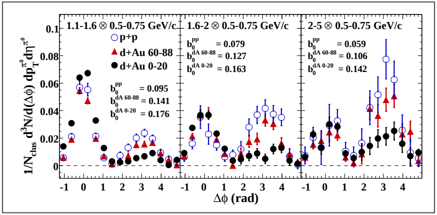
Another example: forward dihadron correlations in dAu

Two particle collision vs. $\Delta\varphi$:

pp peripheral *d*Au central *d*Au



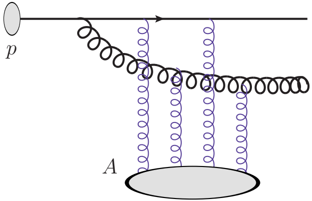
STAR, [arXiv: 1102.0931]



PHENIX, [arXiv: 1105.5112], PRL

Calculating 2-particle correlation in forward pA

- Quark from p (large x) from pdf, radiate gluon
- Propagate eikonally through target $\Longrightarrow$ Wilson lines $U(\mathbf{x})$
- Need target expectation values of Wilson lines — from JIMWLK



$$\frac{\mathrm{d}\sigma^{qA\to qgX}}{\mathrm{d}^3\mathbf{q}\,\mathrm{d}^3\mathbf{k}} \propto \int_{\mathbf{x},\bar{\mathbf{x}},\mathbf{y},\bar{\mathbf{y}}} e^{-i\mathbf{q}\cdot(\mathbf{x}-\bar{\mathbf{x}})}\, e^{-i\mathbf{k}\cdot(\mathbf{y}-\bar{\mathbf{y}})}\, \mathcal{F}(\bar{\mathbf{x}}-\bar{\mathbf{y}},\mathbf{x}-\mathbf{y})$$

$$\left\langle \hat{Q}(\mathbf{y},\bar{\mathbf{y}},\bar{\mathbf{x}},\mathbf{x})\,\hat{D}(\mathbf{x},\bar{\mathbf{x}}) - \hat{D}(\mathbf{y},\mathbf{x})\hat{D}(\mathbf{x},\bar{\mathbf{z}}) - \hat{D}(\mathbf{z},\bar{\mathbf{x}})\hat{D}(\bar{\mathbf{x}},\bar{\mathbf{y}}) + \dots \right\rangle_{\text{target}}$$

$(\mathbf{z} = z\mathbf{x} + (1-z)\mathbf{y},\ \bar{\mathbf{z}} = z\bar{\mathbf{x}} + (1-z)\bar{\mathbf{y}}.)$

$$\hat{D}(\mathbf{x}-\mathbf{y}) \equiv \frac{1}{N_c}\operatorname{Tr} U(\mathbf{x})U^\dagger(\mathbf{y}) \quad \hat{Q}(\mathbf{x},\mathbf{y},\mathbf{u},\mathbf{v}) \equiv \frac{1}{N_c}\operatorname{Tr} U(\mathbf{x})U^\dagger(\mathbf{y})U(\mathbf{u})U^\dagger(\mathbf{v})$$

3 Gluon saturation and the CGC

Classical field and equation of motion

- We were describing the high energy nucleus as a classical field: A^- $\implies$ Wilson line
- What does this imply for the partonic content of the nucleus?
- The physical picture of "gluons as partons" requires two things
  - Infinite momentum frame: nucleus moving fast
    Also change direction: nucleus moves now in $+z$-direction with large p^+.
    Means classical field is large A^+
  - Light cone gauge: have to gauge transform to $A^+ = 0$
- But let us start with the "classical" part.

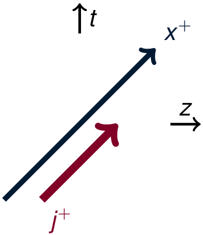
Classical field $\equiv$ from equation of motion

$$[D_\mu, F^{\mu\nu}] = J^\mu$$

What remains is

$$\nabla^2 A^+ = J^+$$

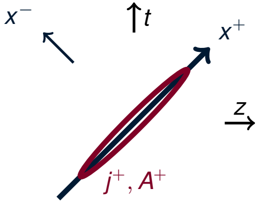
This is nice, the big $+$-field corresponds to a **color** current in the $+$-direction.



Spacetime structure of the field

The current lives on the light cone.

1. Naive explanation: Nucleus is Lorentz-contracted to $\Delta z \sim 2R_A m_A/\sqrt{s}$
2. Real explanation: Current represents large x degrees of freedom
   - They have large p^+, classical field small
   - They are more localised in x^- than the field.



The current is independent of LC time x^+;

glass!

Argument is as above:

1. Time is dilated for the nucleus
2. Any probe will have larger k^- than color current $\implies$ probe will oscillate faster in x^+ and see current as static.

Extreme approximation:

$$j^+(x^-, \mathbf{x}) \approx \delta(x^-)\rho(\mathbf{x})$$
$$A^+(x^-, \mathbf{x}) \approx \delta(x^-)\frac{1}{\nabla^2}\rho(\mathbf{x})$$

Classical field and equation of motion

Now let us gauge transform.

$$A^+ \quad \Rightarrow \quad U^\dagger(\mathbf{x}, x^-) A^+ U(\mathbf{x}, x^-) - \frac{i}{g} U^\dagger(\mathbf{x}, x^-)\partial_- U(\mathbf{x}, x^-) = 0$$

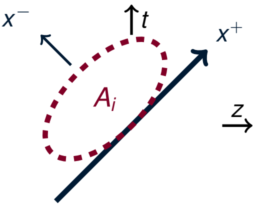
$$A^- \quad \Rightarrow \quad -\frac{i}{g} U^\dagger(\mathbf{x}, x^-)\partial_+ U(\mathbf{x}, x^-) = 0, \text{ still}$$

$$A^i \quad \Rightarrow \quad \frac{i}{g} U^\dagger(\mathbf{x}, x^-)\partial_i U(\mathbf{x}, x^-)$$

transverse pure gauge

This is solved by familiar Wilson line

$$U(\mathbf{x}, x^-) = \mathbb{P} \exp\left[-ig \int^{x^-} \mathrm{d}y^- A^+\right]$$



Now $A^i \sim \theta(x^-)$ — delocalized in x^-, just like small k^+ physical gluons should be.

Weizsäcker-Williams gluon distribution

In LC quantization (Now of nucleus, not γ^*) the number distribution of gluons:

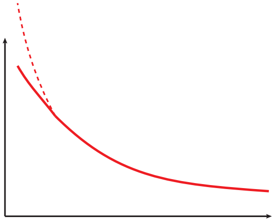
$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{k}\,\mathrm{d}y} \sim \left\langle A^i_a(\mathbf{k}) A^i_a(-\mathbf{k}) \right\rangle$$

- $A^i_a(\mathbf{k})$ is obtained from the Wilson line
- Wilson line is related to DIS dipole cross section, BK equation
- One can express this **Weizsäcker-Williams** gluon distribution as:

$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{k}\,\mathrm{d}y} = \varphi^{\mathrm{WW}}(\mathbf{k}) = \frac{C_\mathrm{F}}{2\pi^3}\frac{1}{\alpha_\mathrm{s}} \int \mathrm{d}^2\mathbf{b} \int \mathrm{d}^2\mathbf{r}\, \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\mathbf{r}^2} \widetilde{\mathcal{N}}(\mathbf{b},\mathbf{r})$$

($\widetilde{\mathcal{N}}$ is the adjoint representation Wilson line correlator)

- Gluon saturation in $\varphi^{\mathrm{WW}}(\mathbf{k})$ at $\mathbf{k} \lesssim Q_\mathrm{s}$
- $\varphi^{\mathrm{WW}}(\mathbf{k}) \sim 1/\alpha_\mathrm{s} \implies$ "**condensate**" of gluons



Now we have a **C**olor **G**lass **C**ondensate.

4 Heavy ion collisions and the glasma initial state

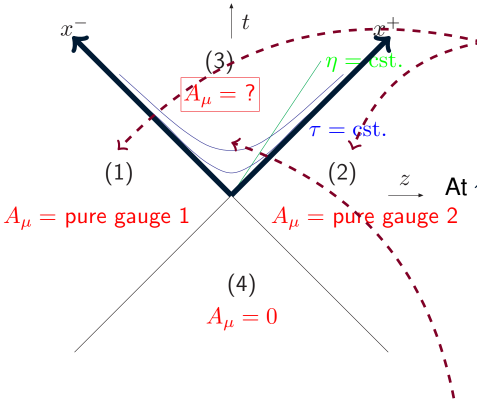
Gluon fields in AA collision

Now two colliding nuclei $\Longrightarrow$ two color currents

$$J^\mu = \delta^{\mu +}\rho_{(1)}(\mathbf{x})\delta(x^-) + \delta^{\mu -}\rho_{(2)}(\mathbf{x})\delta(x^+)$$

Classical **Y**ang-**M**ills

2 pure gauges



$$A^i_{(1,2)} = \frac{i}{g} U_{(1,2)}(\mathbf{x})\partial_i U^\dagger_{(1,2)}(\mathbf{x})$$

$$U_{(1,2)}(\mathbf{x}) = P e^{ig\int dx^- \frac{\rho(\mathbf{x},x^-)}{\nabla^2}}$$

At $\boldsymbol{\tau = 0}$:

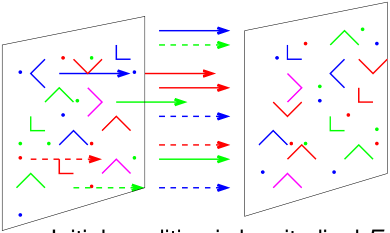
$$A^i\Big|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

$$A^\eta\Big|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

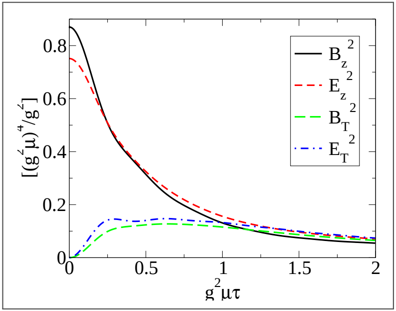
Solve numerically Yang-Mills equations for $\boldsymbol{\tau = \sqrt{2x^+x^-} > 0}$

This is the “glasma” field $\Longrightarrow$ Then average over $\boldsymbol{\rho}$.

Result:



- Initial condition is longitudinal E and B field,
- Depend on transverse coordinate with correlation length $1/Q_s$.
  $\Longrightarrow$ gluon correlations



Gauss law and Bianchi: (here $i = 1 \ldots 3$)

$$[D_i, E^i] = 0, \quad [D_i, B^i] = 0$$

Separate nonabelian parts:

$$\partial_i E^i = ig[A^i, E^i], \quad \partial_i B^i = ig[A^i, B^i]$$

Effective electric and magnetic charge densities.

Deriving the initial condition

Let's work in Fock-Schwinger/temporal gauge $A_\tau = \left(x^+ A^- + x^- A^-\right)/\tau = 0$
$\Longrightarrow$ consistent with LC gauge solutions for both nuclei.

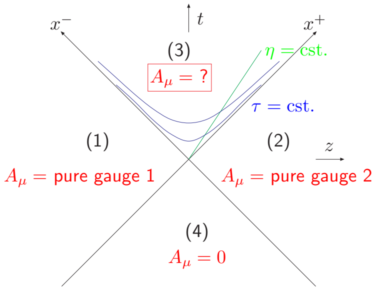
$$\begin{aligned} \text{Ansatz: } A_i &= \overbrace{A_i^{(1)}\theta(-x^+)\theta(x^-) + A_i^{(2)}\theta(x^+)\theta(-x^-)}^{\text{known}} + A_i^{(3)}\theta(x^+)\theta(x^-) \\ A^\pm &= \pm\theta(x^+)\theta(x^-)x^\pm A^\eta \end{aligned}$$

Insert into $[D_\mu, F^{\mu\nu}] = J^\nu$ and match δ-functions

▼

initial condition for region (3):

$$\begin{aligned} A_i^{(3)}|_{\tau=0} &= A_i^{(1)} + A_i^{(2)} \\ A^\eta|_{\tau=0} &= \frac{ig}{2}\left[A_i^{(1)}, A_i^{(2)}\right] \end{aligned}$$



Gluon spectrum

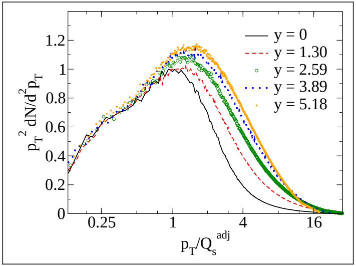
CYM equations can be solved numerically on the lattice.
Decompose solution in Fourier **k**-modes: gluon spectrum

Q_s is only dominant scale

Parametrically $$\frac{dN_g}{dy\, d^2\mathbf{x}\, d^2\mathbf{p}} = \frac{1}{\alpha_s} f\left(\frac{p}{Q_s}\right)$$

Produced gluon spectrum: harder at higher $\sqrt{s}$

(Here: midrapidity, $y \equiv \ln \sqrt{s/s_0}$)



Dilute limit and k_T-factorization

The equations of motion are easy to solve in the **dilute limit**;
(This is a CGC theorist's "pp collision")
Linearized eqations are wave equations

$$\begin{aligned}\left(\tau^2\partial_\tau^2 + \tau\partial_\tau + \tau^2\mathbf{k}^2\right) A_i(\tau,\mathbf{k}) &= 0\\ \left(\tau^2\partial_\tau^2 - \tau\partial_\tau + \tau^2\mathbf{k}^2\right) A_\eta(\tau,\mathbf{k}) &= 0.\end{aligned}$$

$$\Longrightarrow \quad A_i(\tau,\mathbf{k}) = A_i(\tau=0,\mathbf{k})J_0(|\mathbf{k}|\tau) \qquad A^\eta(\tau,\mathbf{k}) = -\frac{1}{\tau|\mathbf{k}|}A^\eta(\tau=0,\mathbf{k})J_1(|\mathbf{k}|\tau).$$

- These are (boost invariant) plane waves $\Longrightarrow$ interpret as particles, gluons.
- Initial fields related to Wilson lines, and via that to the gluon amplitude

Number spectrum **in the dilute limit**: k_T-factorization formula.

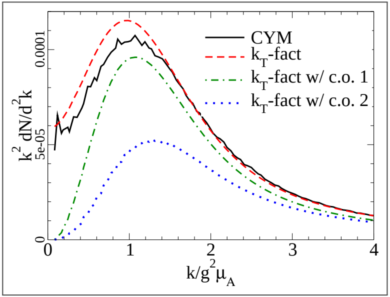
$$\frac{\mathrm{d}N}{\mathrm{d}y\,\mathrm{d}^2\mathbf{k}} = \frac{\alpha_\mathrm{s}}{S_\perp}\frac{2}{C_\mathrm{F}}\frac{1}{k^2}\int \mathrm{d}^2\mathbf{q}\,\varphi^{\mathrm{dip}}(\mathbf{q})\varphi^{\mathrm{dip}}(|\mathbf{k}-\mathbf{q}|).$$

This calculation can also be repeated by assuming that **one** of the two colliding objects is dilute (Theorist's "pA") — **It does not work in "AA"**

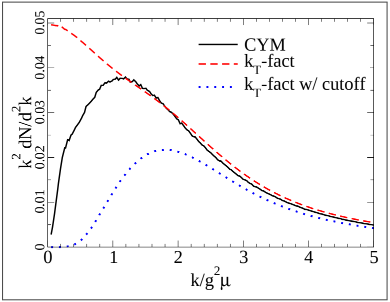
CYM vs. **k**-factorization

- In fact, also in "AA" the k_T-factorization formula works for high p_T
- But it does not give a finite **integrated** total gluon multiplicity,
  - Sometimes this is fixed by an ad hoc cutoff

$$\frac{dN}{d^2\mathbf{p}\,dy} = \frac{1}{\alpha_s}\frac{1}{\mathbf{p}^2}\int_{\mathbf{k}}\left[\theta(p-k)\right]\phi_y(\mathbf{k})\phi_y(\mathbf{p}-\mathbf{k})$$

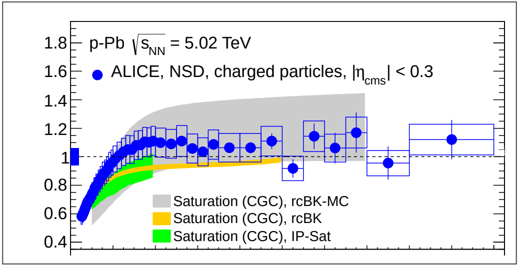


pA: **k**-factorization works



AA: k_T-factorization only for large p_T

Back to R_{pA}



The theory predictions here are calculated with the k_T-factorization formula:

$$\frac{dN}{dy\, d^2\mathbf{k}} = \frac{\alpha_s}{S_\perp} \frac{2}{C_F} \frac{1}{k^2} \int d^2\mathbf{q}\, \varphi^{\mathrm{dip}}(\mathbf{q}) \varphi^{\mathrm{dip}}(|\mathbf{k}-\mathbf{q}|),$$

convoluted with a fragmentation function for $g \to$hadrons.

- You can also rederive the hybrid formula from this, in the asymmetric limit. ($Q_s^A \gg Q_s^p$, i.. $|\mathbf{k}-\mathbf{q}| \gg |\mathbf{q}|$)

Tale of two gluon distributions

This picture has only been clarified recently. One must differentiate

WW distribution

$$\varphi^{\mathrm{WW}}(\mathbf{k}) = \frac{C_\mathrm{F}}{2\pi^3}\frac{1}{\alpha_\mathrm{s}}\int \mathrm{d}^2\mathbf{b}\int \mathrm{d}^2\mathbf{r} \times \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\mathbf{r}^2}\mathcal{N}(\mathbf{b},\mathbf{r})$$

- Comes from actually counting gluons in the nucleus
- Satisfies the usual momentum-space version of the BK equation

Dipole distribution distribution

$$\varphi^{\mathrm{dip}}(\mathbf{k}) = \frac{C_\mathrm{F}}{8\pi^3}\frac{\mathbf{k}^2}{\alpha_\mathrm{s}}\int \mathrm{d}^2\mathbf{b}\int \mathrm{d}^2\mathbf{r} \times e^{i\mathbf{k}\cdot\mathbf{r}}\mathcal{N}(\mathbf{b},\mathbf{r})$$

- Appears in k_T-factorized expression for particle production in pp, pA

(Being careless with $\mathrm{SU}(N_\mathrm{c})$ representations and color factors here.)

Sumamry: timeline of course so far

We have condensed a lot of literature into a short set of lectures:

- Optical diffraction (before 1900)
- quantum mechanical scattering theory (1930's)
- pre-QCD hadron scattering, Regge theory (1960's)
- QCD scattering (1970's)
- large ln s-contributions resummed by BFKL (1980's)
- dipole picture of DIS, BK equation (1990's)
- Classical Yang-Mills description of heavy ions (2000's)