FYSH555, spring 2014

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Office: FL249. No fixed reception hours.

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Part 1: Introduction

Dates, times

- Lectures: Mon, Wed at 14h15, FYS2
- Exercises: Mon, at 16h15, FYS5
  - Problems available online and distributed on Monday lecture
    Return by Monday 10 o'clock to box in lobby **(time may change)**
- Teachers:
  - Tuomas Lappi, part 1: 5 weeks lectures 13.1.–12.2., Room FL249 (next to copy machine)
  - Kari J. Eskola, part 2: 5 weeks lectures 17.2.– (??, or 24.2. ?)
  - Thorsten Renk, part 3: 4 weeks lectures Room FL249 (next to copy machine)
  - Heikki Mäntysaari (part 1 & 3 ?) & Hannu Paukkunen (part 2), exercises

Passing the course:

- Exam: 2/3 of the grade
- Exercises: 1/3 of the grade, 100% done for full points

Contents, outline

Tentative schedule for 1st part: high energy limit of QCD

1. **Preliminaries** Reminder from particle physics (L1)
2. **Small angle scattering** optics — eikonal approx (L2-3), [BP chap 2]
3. **Pre-QCD models** the analytic *S*-matrix, Regge (L4) [FR chap. 1]
4. **The QCD pomeron** gluon exchange, BFKL (L5-6), [BP chap 8, FR chap 3& 4, KL chap 3]
5. **Dipole picture of DIS** (L7), [BP chap 9, FR chap 6, KL chap 4]
6. **Light front quantization** (L8) [KL chap 1]
7. **The BK equation** (L9) [KL chap 4]
8. **Color Glass Condensate** (L10) [KL chap 5]

Rest of course

- Hard processes in QCD: DIS to NLO, parton distributions, DGLAP equation. (K. J. Eskola, 10 lectures)
- Low energy limit of QCD: chiral effective theory (T. Renk, 8 lectures)

Literature, main sources for part 1

Slides and handwritten, scanned, lecture notes will be available on the course web page (links from Korppi)

First 5 weeks follow the books **available in FYS4**

- V. Barone and E. Predazzi, High-Energy Particle Diffraction (Springer 2002) [BP]
- J. R. Forshaw and D. A. Ross, Quantum Chromodynamics and the Pomeron (Cambridge 1997) [FR]
- Yu. Kovchegov & E. Levin: Quantum Chromodynamics at high energy (Cambridge 2012) [KL]

Literature, review articles

- E. Iancu and R. Venugopalan, “The color glass condensate and high energy scattering in QCD,” arXiv:hep-ph/0303204.
- F. Gelis, E. Iancu, J. Jalilian-Marian and R. Venugopalan, “The Color Glass Condensate,” arXiv:1002.0333 [hep-ph].
- F. Gelis, T. Lappi and R. Venugopalan, “High energy scattering in Quantum Chromodynamics,” Int. J. Mod. Phys. E **16** (2007) 2595 [arXiv:0708.0047 [hep-ph]].
- S. J. Brodsky, H. C. Pauli and S. S. Pinsky, “Quantum Chromodynamics and Other Field Theories on the Light Cone,” Phys. Rept. **301** (1998) 299 [arXiv:hep-ph/9705477].
- C. Marquet, “Chromodynamique quantique à haute énergie, théorie et phénoménologie appliqué aux collisions de hadrons,” PhD thesis, in French (!) http://tel.archives-ouvertes.fr/tel-00096416/fr/

Collider experiments: hadronic

- CERN SPS (Super Proton Synchrotron), 1976 –
  - $p\bar{p}$ collider @ $\sqrt{s} = 630\text{GeV} \implies 900\text{GeV}$
  - p, A-fixed target
  - p injector for LHC at 450GeV
  - Experiments: **UA1, UA2**, ... UA9, NAxx, WAxx
- Tevatron: 1983-2011 $p\bar{p}$ @ $\sqrt{s} \approx 1000\text{GeV} \implies 1960\text{GeV}$
- RHIC:
  - pp @ $\sqrt{s} = 500\text{GeV}$
  - $AuAu$ @ $\sqrt{s} = 200A\text{GeV}$
- LHC: 2010 —
  - pp @ $\sqrt{s} = 7\text{TeV} \implies 14\text{TeV}$
  - $AuAu$ @ $\sqrt{s} = 2.76A\text{TeV} \implies 5.5\text{TeV}$

Collider experiments: leptons

Lepton-lepton (less important for this course)

- SLAC SLD: e^+e^- @ $\sqrt{s} = 90\text{GeV}$
- LEP: 1989-2000: e^+e^- @ $\sqrt{s} = 91\text{GeV} \implies 209\text{GeV}$
- ILC $\sqrt{s} \sim \text{TeV}$

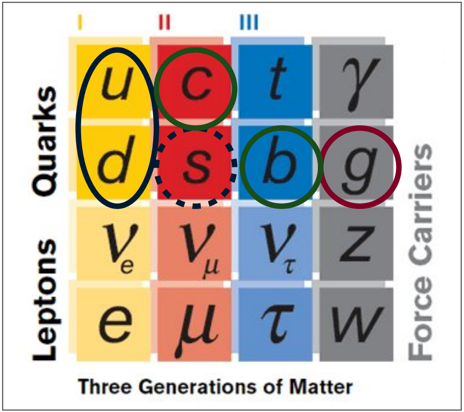
Lepton-proton/hadron **DIS**

- Fixed target, highest energy $2\sqrt{1\text{GeV} \times 465\text{GeV}} = 30\text{GeV}$ at Fermilab muon beam.
- HERA, final $\sim 30\text{GeV}$ on $\sim 900\text{GeV} \implies \sqrt{s} \sim 320\text{GeV}$

Future:

- EIC
  - Electron to collide with RHIC p/A beam 30GeV on 100AGeV / 250GeV $\implies$ $\sqrt{s} \approx 100A\text{GeV}$ / 170GeV
  - p/A beam to collide with JLab 12GeV e^- beam.
- LHeC
  - electron 80GeV ? to collide with LHC. $\sqrt{s_{pe}} = 1.5\text{TeV}$

Elementary particles

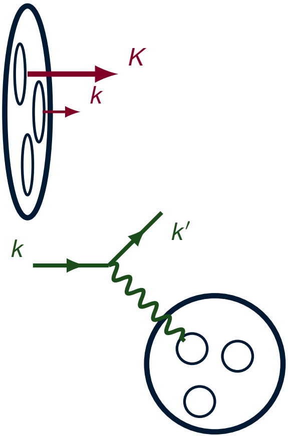


Important in this course

- Light quarks
- Heavy quarks
- gluons

(Top decays fast via weak interaction, different phenomenology)

Partons, IMF



J.D. Bjorken: At high energy hadrons consist of pointlike constituents: “partons”.

- Longit. momentum fraction x: $k = xK$
- Transverse momentum p_T or Q

Measured in **D**eep **I**nelastic **S**cattering

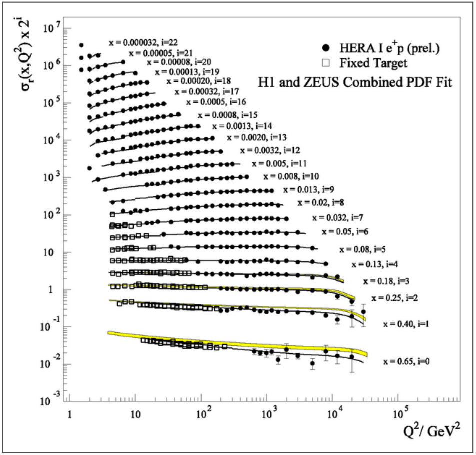
$$\begin{aligned} Q^2 &= -(k-k')^2 = -q^2 \\ x &= \frac{Q^2}{2P\cdot(k-k')} \end{aligned}$$

Bj scaling:

Q^2-dependence is just like for a point-like particle.
Later: partons are quarks and gluons; small deviations from scaling.

Success of partonic picture

Bjorken scaling in inclusive *(ep →anything)* DIS cross section:



- Lines horizontal: free partons.
- Small deviations: understood in pQCD (= perturbative QCD)

Not everything is so simple

Total cross section not calculated from QCD!

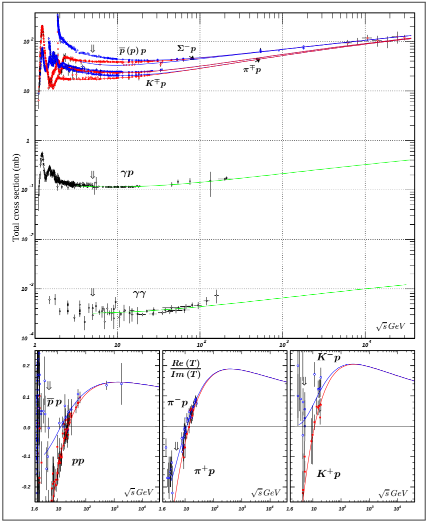
(Why is $\gamma\gamma$ on same plot?

We will learn that γ is a hadron . . .)

$$\frac{\mathrm{Re}(T)}{\mathrm{Im}(T)}$$

matters in this course.

(T = scattering amplitude.)



Reminder from particle physics course

- Cross section, scattering amplitude, Feynman rules
- QCD, color
- Kinematics, rapidity

Cross section and scattering amplitude

- Initial state $|i\rangle$ is a multiparticle **Fock state**
- Time evolution operator from initial to final state is "***S*-matrix**":
  $\Longrightarrow$ final state $S|i\rangle$
- S-matrix element $S_{fi} \equiv \langle f|S|i\rangle$
- Transition probability $P_{fi} = |S_{fi}|^2 \sim$ cross section $i \to f$

It is conventional to separate the case where nothing happens:

$$S = 1 + iT$$

(Separating the i is just a convention)

The 4-momentum is always conserved, it is useful to separate

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^{(4)} \left(\sum p_f - \sum p_i\right) \underbrace{\mathcal{A}(i \to f)}_{\text{scattering amplitude}}$$

Cross section, $2 \to n$ process

Cross section $a + b \to 1 + \cdots + n$

$s \equiv (p_a + p_b)^2$

$$d\sigma = \frac{1}{\Phi} \left| \mathcal{A}(i \to f) \right|^2 d\Pi_n$$

Φ flux factor; $\Phi \approx 2s$ when s is large (masses zero)

$d\Pi_n$ is the n-particle invariant phase space

$$d\Pi_n = \prod_{m=1}^{n} \underbrace{\frac{d^4 p_m}{(2\pi)^4} (2\pi)\delta(p_m^2 - m_m^2)}_{\frac{d^3\mathbf{p}_m}{(2\pi)^3 2E_m}} (2\pi)^4 \delta^{(4)}\left(p_a + p_b - \sum_{i=1}^{n} p_i \right)$$

- Explicitly Lorentz-invariant
- 2π always follows the δ-function
- Cross section is **area**: physical interpretation as probability to scatter integrated over transverse **impact parameter** plane

Feynman rules

To get $i\mathcal{M}$, combine the following (i because $S = 1 + iT$)

1 Propagators

$$\frac{i}{p^2 - m^2 + i\varepsilon}$$

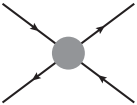
$$\frac{i}{\not{p} - m + i\varepsilon}$$

$$\frac{i g_{\mu\nu}}{p^2 + i\varepsilon}$$

(Feynman gauge)

3 External lines

$u(p)$ $\bar{u}(p)$

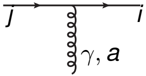


$\bar{v}(p)$ $v(p)$

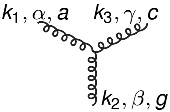
$\varepsilon_\mu(p)_\mu$

$\varepsilon^*_\mu(p)_\mu$

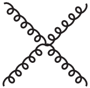
2 Vertices



$$i g t^a_{ij} \gamma^\mu$$



$$g f_{abc}[(k_1 - k_2)^\gamma g^{\alpha\beta} \\ (k_2 - k_3)^\alpha g^{\gamma\beta} \\ (k_3 - k_1)^\beta g^{\alpha\gamma}]$$



$\dots$

4 Internal lines

$$\int \frac{d^4 k}{(2\pi)^4}$$

Gauge theory

QED

$$\text{Lagrangian } \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\psi}\,(i\not\partial + e\not A - m)\,\psi$$

- $\not a = a^\mu \gamma_\mu$
- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (linear in A_μ!)
- $E^i = F^{i0} = F_{0i}$ (sign from $E^i = -\partial_i A^0$)
- $B^i = -\frac{1}{2}\varepsilon^{ijk} F^{jk}$

QCD: add color $N_c = 3$

- $\psi_i,\ i = 1, \ldots, N_c$
- $A^a_\mu,\ a = 1, \ldots, N_c{}^2 - 1 \equiv d_A$

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \overline{\psi_i}\left(i\not\partial \delta_{ij} + e\not A^a t^a_{ij} - m\delta_{ij}\right)\psi_j$$

Matrix notation

Fundamental representation generators:

$$t^a_{ij};\ i,j = 1,\ldots,N_c;\ a = 1,\ldots,{N_c}^2 - 1$$

are **the** ${N_c}^2 - 1$ traceless hermitian $N_c \times N_c$ matrices

Normalization: $$t^a_{ij} t^b_{jk} = \left(\mathrm{Tr}\, t^a t^b\right)_{ik} = \frac{1}{2}\delta_{ik} \quad \text{i.e.} \quad \mathrm{Tr}\, t^a t^b = \frac{1}{2}$$

Structure constants $$[t^a, t^b] = i f^{abc} t^c$$

Matrix notation$A_\mu \equiv A^a_\mu t^a$; $\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\,(i\partial\!\!\!/ + eA\!\!\!/ - m)\,\psi$

Gauge invariance

Field strength tensor $F_{\mu\nu}$ cannot be as simple as QED, but has nonlinear part **gluon-gluon-interaction**

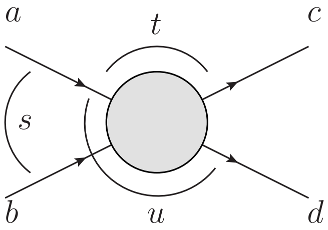
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

Lagrangian is invariant under **gauge transformation**

$$\begin{aligned}
\Omega(x) &: \quad \mathbb{R}^4 \mapsto \mathrm{SU}(N_c) \\
A^\mu(x) &\to \quad \Omega(x) A^\mu(x) \Omega^\dagger(x) + \frac{i}{g} \Omega(x) \partial^\mu \Omega^\dagger(x) \\
\psi(x) &\to \quad \Omega(x) \psi(x)
\end{aligned}$$

$\mathcal{L}$ is invariant (exercise, FYSH300)

Mandelstam variables s, t, u, invariant $\mathrm{d}\sigma / \mathrm{d}t$



(Remember $p_a + p_b = p_c + p_d$)

$$\begin{aligned} s &= (p_a + p_b)^2 = (p_c + p_d)^2 \\ t &= (p_a - p_c)^2 = (p_b - p_d)^2 \\ u &= (p_a - p_d)^2 = (p_b - p_c)^2 \end{aligned}$$

Only 2 independent:

$$s + t + u = m_a^2 + m_b^2 + m_c^2 + m_d^2$$

Mandelstam variables are Lorentz-invariant

You can compute them from 4-vectors in any frame.

Differential cross section in manifestly Lorentz-invariant form

$$\frac{\mathrm{d}\sigma_{ab\to cd}}{\mathrm{d}t} = \frac{|\mathcal{M}_{ab\to cd}|^2}{16\pi s^2} \qquad (m \approx 0)$$

Rapidity

Reminder from particle physics

Successive boosts in one direction with velocity v become easy if we define the **rapidity** ξ as $v = \tanh \xi$. Now for successive boosts $\xi = \xi_1 + \xi_2$. (exercise)

Momentum space rapidity y; $v^z = \tanh y$

$$\begin{aligned} E &= \sqrt{m^2 + p_T^2} \cosh y = m_T^2 \cosh y \qquad \mathbf{p}_T = (p^1, p^2) \\ p^3 &= \sqrt{m^2 + p_T^2} \sinh y = m_T^2 \sinh y \qquad p_T^2 = \mathbf{p}_T^2 \end{aligned}$$

When boosting with a boost ξ this behaves as you would expect . . . (exercise)

Pseudorapidity η is same for $m = 0$: $E = p_T \cosh \eta$, $\quad p^3 = p_T \sinh \eta$

+ Just angle, easy to measure $\Longrightarrow$ experiments use this
- Does not Lorentz-transform easily for massive particles

Light cone variables

For any 4-vector v^μ:

$$v^{\pm} = \frac{1}{\sqrt{2}}\left(v^0 \pm v^3\right)$$

Scalar product

$$\begin{aligned} x \cdot y &= x^0 y^0 - x^3 y^3 - \mathbf{x}_T \cdot \mathbf{y}_T = x^+ y^- + x^- y^+ - \mathbf{x}_T \cdot \mathbf{y}_T \\ v^2 &= (v^0)^2 - (v^3)^2 - \mathbf{v}_T^2 = 2v^+ v^- - \mathbf{v}_T^2 \end{aligned}$$

Thus metric is

$$g_{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = g^{\mu\nu}$$

Raising and lowering indices is not just a sign:

$$v^+ = v_- \qquad v^- = v_+$$

High energy particle traveling in $+$ direction

$$p^+ \approx \sqrt{2}E \quad \gg \quad p^- = \frac{m^2 + {\mathbf{p}_T}^2}{2p^+}$$