FYSH555 spring 2014

Exercise 5, return by Mon Feb 17th at 9.15, discussed Mon Feb 17th, at 12.15 in FYS5

1. Fourier-transform the essential part of the LC wave function for emitting a soft gluon:

$$\int \mathrm{d}^2\mathbf{k}_T e^{i\mathbf{k}_T\cdot\mathbf{r}_T} \frac{\boldsymbol{\varepsilon}_T \cdot \mathbf{k}_T}{\mathbf{k}_T{}^2}.$$

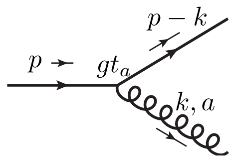
   This can be done analytically (even without mathematica!) by first integrating over the angle, which gives a Bessel function J_1 that is the derivative of J_0; thus the radial integral is easy. Note that there are two independent azimuthal angles, those of $\boldsymbol{\varepsilon}_T$ and $\mathbf{r}_T$. Surprisingly the integral is really convergent without any regularization.

2. (Kovchegov & Levin, 4.5 b) Solve the BK equation in zero transverse dimensions:

$$\partial_y N = \alpha_{\mathrm{s}} N - \alpha_{\mathrm{s}} N^2, \quad N(y=0) = N_0 \ll 1 \tag{1}$$

3. Look at the paper: Phys.Lett. B687 (2010) 174; `arXiv:1001.1378 [hep-ph]` Use INSPIRE to find out how many citations this paper has (quite a lot). How do equations (3) and (4) reduce to the one in the lecture? What is forward rapidity in this case and why is equation (1) supposed to work there better than at midrapidity?

4. (Kovchegov & Levin, exercise 5.1 a,b) Calculate the gluon field radiated from a fast-moving quark using usual covariant theory Feynman rules and the eikonal vertex (which is valid in covariant gauge)



$$A^a_\mu(k) = -igt^a \frac{-ig_{\mu\nu}}{k^2 + i\varepsilon} \bar{u}_\sigma(p-k)\gamma^\nu u_\sigma(p)(2\pi)\delta((p-k)^2) \tag{2}$$

   Remember that at high energy $p^+ \approx (p-k)^+ \gg k^+$. The incoming quark is also on shell, with $p^\mu = (p^+, 0, \mathbf{0}_T)$. Then Fourier-transform

$$A^a_\mu(x) = \int \frac{\mathrm{d}^4k}{(2\pi)^4} e^{-ik\cdot x} A^a_\mu(k) \tag{3}$$

   to get the field in coordinate space

$$A^{+a}_{\mathrm{cov}} = -\frac{g}{\pi} t^a \delta(x^-) \ln|\mathbf{x}_T|\Lambda \tag{4}$$

5. Consider two (independent of each other) transverse ($i,j \in \{1,2\}$) pure gauge fields that depend only on transverse coordinates $A^{(1,2)}_i = A^{(1,2)}_{i,a} t^a = \frac{-i}{g} U(\mathbf{x}_T)\partial_i U^\dagger(\mathbf{x}_T)$. Recall the expression for the field strength tensor $F_{\mu\nu}$ and show that these pure gauges have no longitudinal magnetic field $F^{(1,2)}_{ij} = 0$. Then consider a field that is the sum of the two: $A_i = A^{(1)}_i + A^{(2)}_i$: what is its magnetic field F_{ij}?